Date: Block: Name: #

Simplifying Trig Expressions Using Identities

1. Factor each of the following:

A) $1-\cos^2\theta$ B) $1-\sin^2\theta$ C) $\sin^2\theta-\cos^2\theta$

D) $\sin\alpha-\sin^2\alpha$ E) $\tan^2\alpha-\cot^2\alpha$ F) $\sec^2\theta-1$

2. Express each of the following in terms of $\sin\theta$ or $\cos\theta$, or both.

A) $\dfrac{1}{\sec\theta}$ B) $\sin^2\theta+\dfrac{1}{\sec^2\theta}$ C) $\cos\theta\dfrac{1}{\sec\theta}$

D) $\tan\theta\cos\theta$ E) $1-\csc^2\theta$ F) $\dfrac{1+\cot^2\theta}{\cot^2\theta}$

3. Write each of the following in terms of $\cos\theta$.

A) $\sin^2\theta$ B) $\cos^2\theta$ C) $\cot^2\theta$ D) $\tan^2\theta$

E) $\cot\theta\sin\theta$ F) $\tan\theta\sin\theta$ G) $\dfrac{\csc\theta}{\cot^2\theta}$ H) $\cot\theta\csc\theta$

4. Express each of the following in terms of $\sin\theta$ or $\cos\theta$, or both.

A) $\tan\theta\sec\theta$ B) $\dfrac{\csc^2\theta}{\cot^2\theta}$ C) $1+\tan^2\theta$ D) $1+\cot^2\theta$

E) $\dfrac{\tan\theta}{1+\tan\theta}$ F) $\dfrac{\cot\theta}{1+\cot\theta}$ G) $\sec^2\theta-\tan^2\theta$ H) $\dfrac{1+\tan\theta}{\sec\theta}$

5. Write each of the following in simpler form (this means to write in terms of one of the primary ratios)

A) $\sin\theta\cot\theta$ B) $\sin\theta+\dfrac{\cos^2\theta}{\sin\theta}$ C) $\sin^2\theta+\sin^2\theta\cot^2\theta$

D) $\cos^3\theta+\sin^2\theta\cos\theta$ E) $\cot\theta\sec\theta\sin\theta$ F) $(\sin\theta+\cos\theta)^2-2\sin\theta\cos\theta$

6. Write each of the following in simpler form (this means to write in terms of one of the primary ratios)

A) $\sin\left(x+\dfrac{\pi}{3}\right)-\cos\left(x+\dfrac{\pi}{6}\right)$ B) $\sin\left(x+\dfrac{\pi}{6}\right)+\cos\left(x+\dfrac{\pi}{3}\right)$

C) $\sin\left(\dfrac{3x}{7}\right)\cos\left(\dfrac{4x}{7}\right)+\cos\left(\dfrac{3x}{7}\right)\sin\left(\dfrac{4x}{7}\right)$ D) $\cos(a-b)-\cos(a+b)$

e) $\sin(a+\pi)-\sin(\pi-a)$

Simplifying Trig Expressions Using Identities

① (A) $1-\cos^2\theta$
$=(1+\cos\theta)(1-\cos\theta)$

(B) $1-\sin^2\theta$
$(1+\sin\theta)(1-\sin\theta)$

c) $\sin^2\theta-\cos^2\theta$
$=(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)$

d) $\sin\alpha-\sin^2\alpha$
$=\sin\alpha(1-\sin\alpha)$

e) $\tan^2\alpha-\cot^2\alpha$
$(\tan\alpha+\cot\alpha)(\tan\alpha-\cot\alpha)$

f) $\sec^2\theta-1$
$=(\sec\theta+1)(\sec\theta-1)$

② A) $\frac{1}{\sec\theta}=\cos\theta$

B) $\sin^2\theta+\frac{1}{\sec^2\theta}=\sin^2\theta+\cos^2\theta$

c) $\cos\theta\cdot\frac{1}{\sec\theta}=\cos\theta\cdot\cos\theta$
$=\cos^2\theta$

d) $\frac{\tan\theta\cdot\cos\theta}{\frac{\sin\theta}{\cos\theta}\cdot\cos\theta}=\sin\theta$

e) $1-\csc^2\theta$
$1-\frac{1}{\sin^2\theta}$
$=\frac{\sin^2\theta-1}{\sin^2\theta}=\frac{-\cos^2\theta}{\sin^2\theta}$

f) $\frac{1+\cot^2\theta}{\cot^2\theta}=\frac{1}{\cot^2\theta}+1$
$=\tan^2\theta+1=\frac{\sin^2\theta}{\cos^2\theta}+1$
$=\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}$

③ A) $\sin^2\theta$
$=1-\cos^2\theta$

B) $\cos^2\theta=\cos\theta\cdot\cos\theta$

c) $\cot^2\theta=\frac{\cos^2\theta}{\sin^2\theta}$

d) $\tan^2\theta=\frac{\sin^2\theta}{\cos^2\theta}$

e) $\cot\theta\cdot\sin\theta$
$=\frac{\cos\theta}{\sin\theta}\cdot\sin\theta$
$=|\cos\theta|$

f) $\tan\theta\sin\theta$
$=\frac{\sin^2\theta}{\cos\theta}=\frac{1-\cos^2\theta}{\cos\theta}$

g) $\frac{\csc\theta}{\cot^2\theta}$

$= \frac{1}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{\sin\theta}{\cos^2\theta}$

h) $\cot\theta\csc\theta$

$= \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta}$

$= \frac{\cos\theta}{\sin^2\theta} = \frac{\cos\theta}{1-\cos^2\theta}$

(4) A) $\tan\theta \cdot \sec\theta$

$= \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$

$= \frac{\sin\theta}{\cos^2\theta}$

B) $\frac{\csc^2\theta}{\cot^2\theta} = \frac{1}{\sin^2\theta} \cdot \frac{\sin^2\theta}{\cos^2\theta}$

$= \sin^2\theta$

c) $\cot^2\theta$

$= \frac{\cos^2\theta}{\sin^2\theta}$

d) $\tan^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta}$

e) $\frac{\tan\theta}{1+\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin\theta}{\cos\theta}}$

$\rightarrow \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta+\sin\theta}{\cos\theta}} = \frac{\sin\theta}{\cos\theta+\sin\theta}$

f) $\frac{\cot\theta}{1+\cot\theta}$

$= \frac{\frac{\cos\theta}{\sin\theta}}{1+\frac{\cos\theta}{\sin\theta}}$

$= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta+\cos\theta}{\sin\theta}}$

$= \frac{\cos\theta}{\sin\theta+\cos\theta}$

g) $\sec^2\theta - \tan^2\theta$

$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{1-\sin^2\theta}{\cos^2\theta}$

h) $\frac{1+\tan\theta}{\sec\theta}$

$= \frac{1}{\sec\theta}(1+\tan\theta)$

$\cos\theta\left(1+\frac{\sin\theta}{\cos\theta}\right)$

$\cos\theta+\sin\theta$

5) A) $\sin\theta \cdot \cot\theta$

$= \sin\theta \cdot \frac{\cos\theta}{\sin\theta}$

$= \cos\theta$

B) $\sin\theta + \frac{\cos^2\theta}{\sin\theta}$

$= \frac{\sin^2\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta} = \frac{1}{\sin\theta} = \csc\theta$

c) $\sin^2\theta + \sin^2\theta\cot^2\theta$

$= \sin^2\theta(1 + \cot^2\theta)$

$= \sin^2\theta(\csc^2\theta)$

$= \sin^2\theta \cdot \left(\frac{1}{\sin^2\theta}\right)$

$= 1$

d) $\cos^3\theta + \sin^2\theta\cos\theta$

$= \cos\theta(\cos^2\theta + \sin^2\theta)$

$= \cos\theta(1)$

$= \cos\theta$

e) $\cot\theta\sec\theta\sin\theta$

$= \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} \cdot \sin\theta$

$= 1$

f) $(\sin\theta+\cos\theta)^2 - 2\sin\theta\cos\theta$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

6.A) $\sin\left(x+\frac{\pi}{3}\right) - \cos\left(x+\frac{\pi}{6}\right)$

$= \left(\sin x\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos x\right) - \left(\cos x\cos\frac{\pi}{6} - \sin x\sin\frac{\pi}{6}\right)$

$= \sin x\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\cos x - \cos x\left(\frac{\sqrt{3}}{2}\right) + \sin x\left(\frac{1}{2}\right)$

$= \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x - \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$

$= \sin x$

B) $\sin\left(x+\frac{\pi}{6}\right) + \cos\left(x+\frac{\pi}{3}\right)$

$= \left(\sin x\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6}\right)\cos x\right) + \left(\cos x\cos\left(\frac{\pi}{3}\right) - \sin x\sin\left(\frac{\pi}{3}\right)\right)$

$= \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x + \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

$= \cos x$

c) $\sin\left(\frac{3x}{7}\right)\cos\left(\frac{4x}{7}\right) + \cos\left(\frac{3x}{7}\right)\sin\left(\frac{4x}{7}\right)$

$= \sin\left(\frac{3x}{7} + \frac{4x}{7}\right)$

$= \sin\left(\frac{7x}{7}\right)$

$= \sin x$

D) $\cos(a-b) - \cos(a+b)$

$= (\cos a\cos b + \sin a\sin b) - (\cos a\cos b - \sin a\sin b)$

$= 2\sin a\sin b$

e) $\sin(a+\pi) - \sin(\pi-a)$

$= (\sin a\cos\pi + \cos a\sin\pi) - (\sin\pi\cos a - \cos\pi\sin a)$

$= (-\sin a + 0) - (0 - (-1)\sin a)$

$= -\sin a - \sin a$

$= -2\sin a$