

## Simplifying Trig Expressions Using Identities

1. Factor each of the following:

A)  $1 - \cos^2 \theta$

B)  $1 - \sin^2 \theta$

C)  $\sin^2 \theta - \cos^2 \theta$

D)  $\sin \alpha - \sin^2 \alpha$

E)  $\tan^2 \alpha - \cot^2 \alpha$

F)  $\sec^2 \theta - 1$

2. Express each of the following in terms of  $\sin \theta$  or  $\cos \theta$ , or both.

A)  $\frac{1}{\sec \theta}$

B)  $\sin^2 \theta + \frac{1}{\sec^2 \theta}$

C)  $\cos \theta \frac{1}{\sec \theta}$

D)  $\tan \theta \cos \theta$

E)  $1 - \csc^2 \theta$

F)  $\frac{1 + \cot^2 \theta}{\cot^2 \theta}$

3. Write each of the following in terms of  $\cos \theta$ .

A)  $\sin^2 \theta$

B)  $\cos^2 \theta$

C)  $\cot^2 \theta$

D)  $\tan^2 \theta$

E)  $\cot \theta \sin \theta$

F)  $\tan \theta \sin \theta$

G)  $\frac{\csc \theta}{\cot^2 \theta}$

H)  $\cot \theta \csc \theta$

4. Express each of the following in terms of  $\sin \theta$  or  $\cos \theta$ , or both.

A)  $\tan \theta \sec \theta$

B)  $\frac{\csc^2 \theta}{\cot^2 \theta}$

C)  $1 + \tan^2 \theta$

D)  $1 + \cot^2 \theta$

E)  $\frac{\tan \theta}{1 + \tan \theta}$

F)  $\frac{\cot \theta}{1 + \cot \theta}$

G)  $\sec^2 \theta - \tan^2 \theta$

H)  $\frac{1 + \tan \theta}{\sec \theta}$

5. Write each of the following in simpler form (this means to write in terms of one of the primary ratios)

A)  $\sin \theta \cot \theta$

B)  $\sin \theta + \frac{\cos^2 \theta}{\sin \theta}$

C)  $\sin^2 \theta + \sin^2 \theta \cot^2 \theta$

D)  $\cos^3 \theta + \sin^2 \theta \cos \theta$

E)  $\cot \theta \sec \theta \sin \theta$

F)  $(\sin \theta + \cos \theta)^2 - 2 \sin \theta \cos \theta$

6. Write each of the following in simpler form (this means to write in terms of one of the primary ratios)

A)  $\sin\left(x + \frac{\pi}{3}\right) - \cos\left(x + \frac{\pi}{6}\right)$

B)  $\sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{3}\right)$

C)  $\sin\left(\frac{3x}{7}\right) \cos\left(\frac{4x}{7}\right) + \cos\left(\frac{3x}{7}\right) \sin\left(\frac{4x}{7}\right)$

D)  $\cos(a - b) - \cos(a + b)$

e)  $\sin(a + \pi) - \sin(\pi - a)$

## Simplifying Trig Expressions Using Identities

$$\textcircled{1} \text{ A) } 1 - \cos^2 \theta \\ = (1 + \cos \theta)(1 - \cos \theta)$$

$$\text{B) } 1 - \sin^2 \theta \\ = (1 + \sin \theta)(1 - \sin \theta)$$

$$\text{c) } \sin^2 \theta - \cos^2 \theta \\ = (\sin \theta + \cos \theta)(\sin \theta - \cos \theta)$$

$$\text{d) } \sin \alpha - \sin^2 \alpha \\ = \sin \alpha (1 - \sin \alpha)$$

$$\text{e) } \tan^2 \alpha - \cot^2 \alpha \\ = (\tan \alpha + \cot \alpha)(\tan \alpha - \cot \alpha)$$

$$\text{f) } \sec^2 \theta - 1 \\ = (\sec \theta + 1)(\sec \theta - 1)$$

$$\textcircled{2} \text{ A) } \frac{1}{\sec \theta} = \cos \theta$$

$$\text{B) } \sin^2 \theta + \frac{1}{\sec^2 \theta} = \sin^2 \theta + \cos^2 \theta$$

$$\text{c) } \cos \theta \cdot \frac{1}{\sec \theta} = \cos \theta \cdot \cos \theta \\ = \cos^2 \theta$$

$$\text{d) } \tan \theta \cdot \cos \theta \\ = \frac{\sin \theta}{\cos \theta} \cdot \cos \theta = \sin \theta$$

$$\text{e) } 1 - \csc^2 \theta \\ = 1 - \frac{1}{\sin^2 \theta} \\ = \frac{\sin^2 \theta - 1}{\sin^2 \theta} = \frac{-\cos^2 \theta}{\sin^2 \theta}$$

$$\text{f) } \frac{1 + \cot^2 \theta}{\cot^2 \theta} = \frac{1}{\cot^2 \theta} + 1 \\ = \tan^2 \theta + 1 = \frac{\sin^2 \theta}{\cos^2 \theta} + 1 \\ = \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\textcircled{3} \text{ A) } \sin^2 \theta \\ = 1 - \cos^2 \theta$$

$$\text{B) } \cos^2 \theta = \cos \theta \cdot \cos \theta$$

$$\text{c) } \cot^2 \theta = \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$\text{d) } \tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\text{e) } \cot \theta \cdot \sin \theta \\ = \frac{\cos \theta}{\sin \theta} \cdot \sin \theta \\ = \underline{\cos \theta}$$

$$\text{f) } \tan \theta \cdot \sin \theta \\ = \frac{\sin^2 \theta}{\cos \theta} = \frac{1 - \cos^2 \theta}{\cos \theta}$$

$$\begin{aligned}
 \text{g) } \frac{\csc \theta}{\cot^2 \theta} &= \frac{1}{\sin \theta} \cdot \frac{\sin^2 \theta}{\cos^2 \theta} \\
 &= \frac{\sin \theta}{\cos^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 \text{h) } \cot \theta \csc \theta &= \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\sin \theta} \\
 &= \frac{\cos \theta}{\sin^2 \theta} = \frac{\cos \theta}{1 - \cos^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 \text{④ A) } \tan \theta \cdot \sec \theta &= \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} \\
 &= \frac{\sin \theta}{\cos^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 \text{B) } \frac{\csc^2 \theta}{\cot^2 \theta} &= \frac{1}{\sin^2 \theta} \cdot \frac{\sin^2 \theta}{\cos^2 \theta} \\
 &= \sin^2 \theta
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } \cot^2 \theta &= \frac{\cos^2 \theta}{\sin^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } \tan^2 \theta &= \frac{\sin^2 \theta}{\cos^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 \text{e) } \frac{\tan \theta}{1 + \tan \theta} &= \frac{\frac{\sin \theta}{\cos \theta}}{1 + \frac{\sin \theta}{\cos \theta}} = \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\cos \theta + \sin \theta}{\cos \theta}} = \frac{\sin \theta}{\cos \theta + \sin \theta}
 \end{aligned}$$

$$\begin{aligned}
 \text{f) } \frac{\cot \theta}{1 + \cot \theta} &= \frac{\frac{\cos \theta}{\sin \theta}}{1 + \frac{\cos \theta}{\sin \theta}} \\
 &= \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta + \cos \theta}{\sin \theta}} = \frac{\cos \theta}{\sin \theta + \cos \theta}
 \end{aligned}$$

$$\begin{aligned}
 \text{g) } \sec^2 \theta - \tan^2 \theta &= \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} \\
 &= \frac{1 - \sin^2 \theta}{\cos^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 \text{h) } \frac{1 + \tan \theta}{\sec \theta} &= \frac{1}{\sec \theta} (1 + \tan \theta) \\
 &= \cos \theta \left( 1 + \frac{\sin \theta}{\cos \theta} \right) \\
 &= \cos \theta + \sin \theta
 \end{aligned}$$

3

$$\begin{aligned} 5) \text{ A) } \sin \theta \cdot \cot \theta & \\ &= \sin \theta \cdot \frac{\cos \theta}{\sin \theta} \\ &= \cos \theta \end{aligned}$$

$$\begin{aligned} \text{B) } \sin \theta + \frac{\cos^2 \theta}{\sin \theta} & \\ &= \frac{\sin^2 \theta}{\sin \theta} + \frac{\cos^2 \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta} = \frac{1}{\sin \theta} = \csc \theta \end{aligned}$$

$$\begin{aligned} \text{c) } \sin^2 \theta + \sin^2 \theta \cot^2 \theta & \\ &= \sin^2 \theta (1 + \cot^2 \theta) \\ &= \sin^2 \theta (\csc^2 \theta) \\ &= \sin^2 \theta \cdot \left( \frac{1}{\sin^2 \theta} \right) \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{d) } \cos^3 \theta + \sin^2 \theta \cos \theta & \\ &= \cos \theta (\cos^2 \theta + \sin^2 \theta) \\ &= \cos \theta (1) \\ &= \cos \theta \end{aligned}$$

$$\begin{aligned} \text{e) } \cot \theta \sec \theta \sin \theta & \\ &= \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} \cdot \sin \theta \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{f) } (\sin \theta + \cos \theta)^2 - 2 \sin \theta \cos \theta & \\ &= \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta - 2 \sin \theta \cos \theta \\ &= \sin^2 \theta + \cos^2 \theta \\ &= 1 \end{aligned}$$

$$\begin{aligned}
\text{A) } & \sin\left(x + \frac{\pi}{3}\right) - \cos\left(x + \frac{\pi}{6}\right) \\
&= \left(\sin x \cos \frac{\pi}{3} + \sin \frac{\pi}{3} \cos x\right) - \left(\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6}\right) \\
&= \sin x \left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) \cos x - \cos x \left(\frac{\sqrt{3}}{2}\right) + \sin x \left(\frac{1}{2}\right) \\
&= \frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x - \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x \\
&= \sin x
\end{aligned}$$

$$\begin{aligned}
\text{B) } & \sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{3}\right) \\
&= \left(\sin x \cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6}\right) \cos x\right) + \left(\cos x \cos\left(\frac{\pi}{3}\right) - \sin x \sin\left(\frac{\pi}{3}\right)\right) \\
&= \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x + \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x \\
&= \cos x
\end{aligned}$$

$$\begin{aligned}
\text{C) } & \sin\left(\frac{3x}{7}\right) \cos\left(\frac{4x}{7}\right) + \cos\left(\frac{3x}{7}\right) \sin\left(\frac{4x}{7}\right) \\
&= \sin\left(\frac{3x}{7} + \frac{4x}{7}\right) \\
&= \sin\left(\frac{7x}{7}\right) \\
&= \sin x
\end{aligned}$$

$$\begin{aligned}
\text{D) } & \cos(a-b) - \cos(a+b) \\
&= (\cos a \cos b + \sin a \sin b) - (\cos a \cos b - \sin a \sin b) \\
&= 2 \sin a \sin b
\end{aligned}$$

$$\begin{aligned}
\text{E) } & \sin(a+\pi) - \sin(\pi-a) \\
&= (\sin a \cos \pi + \cos a \sin \pi) - (\sin \pi \cos a - \cos \pi \sin a) \\
&= (-\sin a + 0) - (0 - (-1) \sin a) \\
&= -\sin a - \sin a \\
&= -2 \sin a
\end{aligned}$$