Date: ____________ Block: ________ NAME: ______________________________

Proving Trigonometric Identities

Prove the following identities:

1. $\dfrac{1+\cos x}{\cos x} = \dfrac{\tan^2 x}{\sec x - 1}$

2. $\dfrac{\cos^2\theta + \tan^2\theta - 1}{\sin^2\theta} = \tan^2\theta$

3. $\sin(x+y+z) = \sin x\cos y\cos z + \cos x\sin y\cos z + \cos x\cos y\sin z - \sin x\sin y\sin z$

4. $\cot(x-y) = \dfrac{\cot x\cot y + 1}{\cot y - \cot x}$

5. $\dfrac{\sin 4x}{\sin x} = 4\cos x\cos 2x$

6. $\dfrac{1+\sin 2x}{\sin 2x} = 1 + \dfrac{1}{2}\sec x\csc x$

7. $\cos^4\theta - \sin^4\theta = \cos 2\theta$

8. $\cos(3x) = 4\cos^3 x - 3\cos x$

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Proving Trig Identities

A) $\dfrac{1+\cos x}{\cos x} = \dfrac{\tan^2 x}{\sec x - 1}$

$$\frac{\sec^2 x - 1}{\sec x - 1}$$

$$\frac{(\sec x - 1)(\sec x + 1)}{(\sec x - 1)}$$

$$\sec x + 1$$

$$\frac{1}{\cos x} + 1$$

$$\frac{1}{\cos x} + \frac{\cos x}{\cos x}$$

$$\frac{1+\cos x}{\cos x}$$

2) $\dfrac{\cos^2\theta + \tan^2\theta - 1}{\sin^2\theta} = \tan^2\theta$

$\dfrac{\cos^2\theta - 1 + \tan^2\theta}{\sin^2\theta}$

$\dfrac{-\sin^2\theta + \tan^2\theta}{\sin^2\theta}$

$-1 + \left(\dfrac{\sin^2\theta}{\cos^2\theta}\right)\left(\dfrac{1}{\sin^2\theta}\right)$

$-1 + \dfrac{1}{\cos^2\theta}$

$-1 + \sec^2\theta$

$\tan^2\theta$

(3)

3) $\sin(x+y+z) = \sin x \cos y \cos z + \cos x \sin y \cos z + \cos x \cos y \sin z - \sin x \sin y \sin z$

Left hand side:

$\sin((x+y)+z)$

$\sin(x+y)\cos z + \sin z \cos(x+y)$

$(\sin x \cos y + \sin y \cos x)\cos z + \sin z(\cos x \cos y - \sin x \sin y)$

$\sin x \cos y \cos z + \sin y \cos x \cos z + \sin z \cos x \cos y - \sin x \sin y \sin z$

$= RHS.$

4)

(4) $\cot(x-y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$

$\dfrac{\cos(x-y)}{\sin(x-y)}$

$\dfrac{\cos x \cos y + \sin x \sin y}{\sin x \cos y - \cos x \sin y}$

$\Rightarrow$ multiply numerator + denominator by $\dfrac{1}{\sin x \sin y}$

$\dfrac{\dfrac{\cos x \cos y}{\sin x \sin y} + \dfrac{\sin x \sin y}{\sin x \sin y}}{\dfrac{\sin x \cos y}{\sin x \sin y} - \dfrac{\cos x \sin y}{\sin x \sin y}}$

$\dfrac{\cot x \cot y + 1}{\cot y - \cot x}$

5) $\frac{\sin 4x}{\sin x} = 4\cos x \cos 2x$

$\frac{\sin(2x+2x)}{\sin x}$

$\frac{\sin 2x\cos 2x + \sin 2x\cos 2y}{\sin x}$

$\frac{2\sin 2x\cos 2x}{\sin x}$

$\frac{2(2\sin x\cos x)\cos 2x}{\sin x}$

$4\cos x\cos 2x$

(6) $\frac{1+\sin 2x}{\sin 2x} = 1 + \frac{1}{2}\sec x \csc x$ (6)

$\frac{1}{\sin 2x} + \frac{\sin 2x}{\sin 2x}$

$\frac{1}{2\sin x\cos x} + 1$

$1 + \left(\frac{1}{2}\right)\left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x}\right)$

$1 + \frac{1}{2}\csc x \sec x$

⑦ $\cos^4\theta - \sin^4\theta = \cos 2\theta$

$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$(\cos^2\theta - \sin^2\theta)(1)$

$\cos 2\theta$

⑧ $\cos(3x) = 4\cos^3 x - 3\cos x$

$\cos(2x+x)$

$\cos 2x \cos x - \sin 2x \sin x$

$(2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x$

$2\cos^3 x - \cos x - 2\sin^2 x \cos x$

$\cos x(2\cos^2 x - 1 - 2\sin^2 x)$

$\cos x(2\cos^2 x - 1 - 2(1-\cos^2 x))$

$\cos x(2\cos^2 x - 1 - 2 + 2\cos^2 x)$

$\cos x(4\cos^2 x - 3)$

$4\cos^3 x - 3\cos x$