Pre Calculus Worksheet Rational Root Theorem and Factoring

1. Factor the following polynomials completely. Solve for all roots.

   (a) $y = x^3+x^2-9x+7$

   (b) $y = x^4-x^3-4x^2+2x+4$

   (c) $y = 2x^5- x^4 - 26x^3+ 13x^2+72x - 36$

   (d) $y = 12x^2 - 28x + 15$

   (e) $y = 21 x^2 + 11x - 40$

   (f) $y = 2x^4 + 5 x^3- x^2+ 5x - 3$

2. List the possible rational zeros of each function. Then determine all the roots (both rationals and irrationals) and identify any double or triple roots.

   (a) $y = x^3-5x^2+17x-13$

   (b) $y = 4x^3+12x^2+x+3$

   (c) $y = 3x^3+10x^2+4x-8$

   (d) $y = 30x^3-x^2-6x+1$

   (e) $y = x^4-2x^3-21x^2+22x+40$

3. Determine the value of m so that $x+2$ is a factor of $x^3 + x^2 + mx +6$.

4. Determine the value of n so that $x-2$ is a factor of $x^3 - 3x + n$

5. Find the values of a and b if $ax^3 + bx^2 + 3x - 4$ has a remainder of -2 when divided by $(x-1)$ and a remainder of 2 when divided by $(x-2)$.

6. When a polynomial $P(x)$ is divided by $(2x-1)$ the quotient is x^2-2x-1 and the remainder is -4. Find $P(x)$.

1A) $y = x^3 + x^2 - 9x + 7$ $\quad a = \pm 1, \ b = \pm 1, \pm 7$

$x = 1$ is a root

1	1	1	-9	7
	↓	1	2	-7
	1	2	-7	0

$(x-1)(x^2+2x-7)$

↳ Does not factor

B) $y = x^4 + x^3 - 4x^2 + 2x + 4$ $\quad a = \pm 1, \ b = \pm 1, \pm 2, \pm 4$

$x = -1$ is a root

-1	1	-1	-4	2	4
	↓	-1	2	2	-4
	1	-2	-2	4	0

$x = 2$ is a root

2	1	-2	-2	4
	↓	2	0	4
	1	0	-2	0

$(x+1)(x-2)(x^2-2)$

↳ does not factor

C) $y = \underbrace{2x^5 - x^4}_{x^4(2x-1)} - \underbrace{26x^3 + 13x^2}_{-13x^2(2x-1)} + \underbrace{72x - 36}_{+36(2x-1)}$

$(2x-1)(x^4 - 13x^2 + 36)$

$(2x-1)(x^2-9)(x^2-4)$

$(2x-1)(x+3)(x-3)(x+2)(x-2)$

D) $y = 12x^2 - 28x + 15$

$= \underbrace{12x^2 - 10x}_{2x(6x-5)} - \underbrace{18x + 15}_{-3(6x-5)}$

$= (6x-5)(2x-3)$

$12 \times 15 = 180$

$\_\_ \times \_\_ = 180$

$\_\_ + \_\_ = -28$

E) $y = 21x^2 + 11x - 40$

$= \underbrace{21x^2 + 35x}_{7x(3x+5)} - \underbrace{24x - 40}_{-8(3x+5)}$

$= (3x+5)(7x-8)$

$21 \times -40 = -840$

$\_\_ \times \_\_ = -840$

$\_\_ + \_\_ = 11$

1f) $y = 2x^4 + 5x^3 - x^2 + 5x - 3$ $\quad a = \pm 1, \ b = \pm 1, \pm 3$

$x = -3$ is a root

-3	2	5	-1	5	-3
	↓	-6	3	-6	3
	2	-1	2	-1	0

$(x+3)(2x^3 - 1x^2 + 2x - 1)$

$(x+3)[x^2(2x-1) + (2x-1)]$

$(x+3)(2x-1)(x^2+1)$

↳ does not factor

2) A) $y = x^3 - 5x^2 + 17x - 13$ $\quad a = \pm 1 \quad b = \pm 1, \pm 13$

$x = 1$ is a root

1	1	-5	17	-13
	↓	1	-4	13
	1	-4	13	0

$(x-1)(x^2 - 4x + 13)$

↳ does not factor

B) $y = \underbrace{4x^3 + 12x^2} + \underbrace{x + 3}$

$= 4x^2(x+3) + 1(x+3)$

$= (x+3)(4x^2+1)$

↳ does not factor

c) $y = 3x^3 + 10x^2 + 4x + 8$ $\quad a = \pm 1, \pm 3$

$b = \pm 1, \pm 2, \pm 4, \pm 8$

↑

Does not factor ∴

2d) $y = 30x^3 - x^2 - 6x + 1$

$a = \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

$b = \pm 1$

$x = -\frac{1}{2}$ is a root

$-\frac{1}{2}$	30	−1	−6	1
	↓	−15	8	−1
	30	−16	2	0

$(x + \frac{1}{2})(30x^2 - 16x + 2)$

$\_x\_ = 60$

$\_+\_ = -16$

$(x + \frac{1}{2})(30x^2 - 10x - 6x + 2)$

$(x + \frac{1}{2})(10x(3x - 1) - 2(3x - 1))$

$(x + \frac{1}{2})(3x - 1)(10x - 2)$

$(x + \frac{1}{2})(3x - 1)(2)(5x - 1)$

combine

$(2x + 1)(3x - 1)(5x - 1)$

e) $y = x^4 - 2x^3 - 21x^2 + 22x + 40$

$a = \pm 1$

$b = \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

$x = 1$ is a root

$x = 2$ is a root

−1	1	−2	−21	22	40
	↓	−1	3	+18	−40
	1	−3	−18	40	0

2	1	−3	−18	40
	↓	2	−2	−40
	1	−1	−20	0

$(x+1)(x-2)(x^2 - x - 20)$

$(x+1)(x-2)(x-5)(x+4)$

3) $P(x)=x^3+x^2+mx+b$ $x+2$ is a factor $\therefore P(-2)=0$

$0=(-2)^3+(-2)^2+m(-2)+b$
$0=-8+4-2m+b$
$0=2-2m$
$2m=2$
$m=1$

4) $f(x)=x^3-3x+n$ $x-2$ is a factor $\therefore f(2)=0$

$0=(2)^3-3(2)+n$
$0=8-6+n$
$-2=n$

5) $f(x)=ax^3+bx^2+3x-4$

$f(1)=-2$
$-2=a(1)^3+b(1)^2+3(1)-4$
$-2=a+b-1$
$-1=a+b$ (2)

$f(1)=-2$
$f(2)=2$
$f(2)=2$
$2=a(2)^3+b(2)^2+3(2)-4$
$2=8a+4b+6-4$
$2=8a+4b+2$
$0=8a+4b$
$0=2a+b$
$-2a=b$ (1)

$-1=a+(-2a)$
$-1=-a$
$1=a$ $\therefore$ $b=-2$

6) $P(x) \div (2x-1) = x^2 - 2x - 1 + \frac{-4}{2x-1}$

$$\frac{P(x)}{2x-1} = x^2\left(\frac{2x-1}{2x-1}\right) - 2x\left(\frac{2x-1}{2x-1}\right) - 1\left(\frac{2x-1}{2x-1}\right) - \frac{4}{2x-1}$$

$$= \frac{2x^3 - x^2 - 4x^2 + 2x - 2x + 1 - 4}{2x-1}$$

$$= \frac{2x^3 - 5x^2 - 3}{2x-1}$$

$\therefore P(x) = 2x^3 - 5x^2 - 3$