7e) $49^x + 1 = 2(7^x)$

$(7^2)^x + 1 = 2(7^x)$

$(7^x)^2 + 1 = 2(7^x)$

$(7^x)^2 - 2(7^x) + 1 = 0$ let $m = 7^x$

$m^2 - 2m + 1 = 0$

$(m-1)(m-1) = 0$

$m - 1 = 0$

$m = 1$

$7^x = 1$

$7^x = 7^0$

$x = 0$

$$\left((-7)^0 + 5x^{-2}\right)^{-3}$$

$$\left(1 + \frac{5}{x^2}\right)^{-3}$$

$$\left(\frac{x^2}{x^2} + \frac{5}{x^2}\right)^{-3}$$

$$\left(\frac{x^2+5}{x^2}\right)^{-3}$$

$$\left(\frac{x^2}{x^2+5}\right)^{3} = \boxed{\frac{x^6}{(x^2+5)^3}}$$ → have to leave like this

Geometric Sequence makes an exponential graph

$$\{1, 2, 4, 8, 16, \ldots\}$$

any term divided by the previous term gives a common ratio

$$\frac{2}{1} = 2 \quad \frac{4}{2} = 2 \quad \frac{8}{4} = 2 \quad \frac{16}{8} = 2$$

$$y = a(c)^{b(x-h)} + K$$

$c = 2$ (base of exponential equation)

rate of increase or decrease

examples of c value from word problems

15% decrease = 0.85

double = 2

half life = 1/2

15% increase = 1.15

$y = a(c)^{b(x-h)} + K$

K = H.A (can't go below)

HS = how often does the rate of change apply.

ex. doubles every 4 hours

HS = 4
in eqn = 1/4

$y = 2^{x/4}$

$a + K$ = initial value (y int)

only for word problems with no HT

The y int for an equation with a HT is NOT $a+k$

Normally to find y int set $x=0$ and solve for y.

Applications of Exponential Equations

1. The half-life of radon is 92 hours. If the initial amount was 48 g, how long will it take for the radon to decay to 3 g?

$C = \frac{1}{2}$

$HS = 92hrs$

$a + K = 48$

$a + 0 = 48$

$a = 48$

$K = H.A$

$48, 24, 12 \ldots$

$K = 0$

$y = 48\left(\frac{1}{2}\right)^{x/92}$

$3 = 48\left(\frac{1}{2}\right)^{x/92}$

$\frac{3}{48} = \left(\frac{1}{2}\right)^{x/92}$

$\frac{1}{16} = \left(\frac{1}{2}\right)^{x/92}$

$\left(\frac{1}{2}\right)^{4} = \left(\frac{1}{2}\right)^{x/92}$

$4 = \frac{x}{92} \rightarrow 368 = x$

15 days
8 hours

3. A cup of hot chocolate is served at an initial temperature of 80°C and then allowed to cool in a stadium with an air temperature of 5°C. The difference between the hot chocolate temperature and the temperature of the stadium will decrease by 30% every six minutes. If T represents the temperature of the hot chocolate in degrees Celsius, measured as a function of time, t, in minutes

A) give the equation of the relationship between temperature and time in the form

$$T = a(c)^{b(t-h)} + k$$

HS = 6 $\quad$ c = 0.7

$a + k = 80$

$a + 5 = 80$

$a = 75$

K = lowest value is 5°

K = 5

$$T = 75(0.7)^{t/6} + 5$$

B) What is the temperature after 11 minutes?

$$T = 75(0.7)^{11/6} + 5$$

$$T = 44^\circ$$

C) How long does it take the hot chocolate to cool to a temperature of 40°C?

$$40 = 75(0.7)^{t/6} + 5$$

$y_1 = 40$ and $y_2 = 75(0.7)^{t/6} + 5$

Find the intersection pt → 2^{nd} calc

intersect

move cursor

enter

(13.91, 40)

13.91 minutes

13 min 55 seconds

2. Compound Interest

Balance – the money you have in the bank.

Principal – the balance on which the bank pays you interest

starting value

Compound Interest - After a set period of time, the interest is added to your account – then the next lot of interest is calculated on the higher balance.

Consider an investment of $500 with an interest rate of 7% per annum paid each year and compounded annually.

per year

After year	Interest paid	Account balance
0	—	500
1	7% of 500 = 35	535
2	7% of 535 = 37.45	535 + 37.45 = 572.45
3	7% of 572.45 = 40.07	612.52

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

A = the amount of money at the end of the investment
P = the principle amount invested (start)
r is the interest rate
n is the number of compounding periods per year

t = in years.

yearly $n=1$
monthly $n=12$
weekly $n=52$
quarterly $n=4$
semi annually $n=2$
semi monthly $n=24$
bi monthly (every 2 months) $n=6$

Example:

If I am able to invest 7.6% p.a. compounded semi-annually, how much should I invest NOW to achieve a maturing value of $10000 in 5 years time?

$n=2$ $\quad r=0.076$ $\quad A=10000$

$t=5$

$P=?$

$$10000 = P\left(1+\frac{0.076}{2}\right)^{2(5)}$$

$$10000 = P(1.038)^{10}$$

$$10000 = P(1.452023132)$$

$$\$6886.94 = P$$

HW
pg 364
9, 10, 12, 14, 16, 17