7.3

Solving Exponential Equations

Focus on...

- determining the solution of an exponential equation in which the bases are powers of one another
- solving problems that involve exponential growth or decay
- solving problems that involve the application of exponential equations to loans, mortgages, and investments

$x^1 = x$

$x^0 = 1$

$x^{-m} = \frac{1}{x^m}$

$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

$x^m \cdot x^n = x^{m+n}$ (only works with same base)

$\frac{x^m}{x^n} = x^{m-n}$ (same base)

$(x^m)^n = x^{m \cdot n}$

$$\left.\begin{array}{l} x^{m/n} = \sqrt[n]{x^m} \\ x^{m/n} = \left(\sqrt[n]{x}\right)^m \end{array}\right\}$$

ex. $\sqrt{x} = x^{1/2}$

$\sqrt[3]{x} = x^{1/3}$

$\sqrt[3]{x^2} = x^{2/3}$

$\left(\sqrt[6]{x}\right)^8 = x^{6/8}$

$(xy)^m = x^m y^m$

$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

Review of Laws of Exponents:

1. Simplify each expression. Leave only positive exponents in your answer.

A) $\dfrac{(x^n)^2(x^{n+2})^2}{x^n}$

$$= \frac{\left(x^{2n}\right)\left(x^{2(n+2)}\right)}{x^n}$$

$$= \frac{x^{2n}\,x^{2n+4}}{x^n} = \frac{x^{4n+4}}{x^n} = x^{3n+4}$$

C) $\left(\sqrt{(a^{\frac{2}{3}})}\right)^{-3}$

$$\left(\left(a^{2/3}\right)^{1/2}\right)^{-3}$$

$$\left(a^{2/3 \cdot \frac{1}{2}}\right)^{-3}$$

$$\left(a^{1/3}\right)^{-3}$$

$$a^{-1} = \boxed{\frac{1}{a}}$$

D) $(2\sqrt[3]{c})\left(\frac{1}{\sqrt{c}}\right)$

$$2(c)^{1/3} \cdot \frac{1}{c^{1/2}}$$

$$2(c)^{1/3} \cdot c^{-1/2}$$

$$2\left(c^{\frac{1}{3} + -\frac{1}{2}}\right)$$

$$2\left(c^{2/6 + -3/6}\right)$$

$$2\left(c^{-1/6}\right)$$

$$2(c)^{-1/6}$$

$$2 \cdot \frac{1}{c^{1/6}}$$

$$\boxed{\frac{2}{c^{1/6}}}$$

2. Evaluate each expression:

B) $(-6)^0$

1

C) -6^0

-1

D) $\frac{5^{-3}}{5^{-4}}$

$5^{-3-(-4)}$

5^1

5

D) $\frac{5^{-3}}{5^{-4}} = \frac{5^4}{5^3} = 5^1 = 5$

3. Express each as a power of 2 and evaluate if possible.

A) $(4^2)^3$

$((2^2)^2)^3$

$2^{2\cdot 2\cdot 3}$

$\boxed{2^{12}}$

B) $(4^{m+3})(2)$

$(2^2)^{m+3} \cdot 2$

$2^{2m+6} \cdot 2^1$

$\boxed{2^{2m+7}}$

4. Write each as a single power.

A) $\dfrac{(8^{2n+1})(4^{2-n})}{(2^{2n})^3}$

$$= \frac{(2^3)^{2n+1} \cdot (2^2)^{2-n}}{2^{6n}}$$

$$= \frac{2^{6n+3} \cdot 2^{4-2n}}{2^{6n}}$$

$$= 2^{4n+7} \div 2^{6n}$$

$$\frac{2^{4n+7}}{2^{6n}}$$

$$2^{(4n+7)-6n}$$

$$2^{-2n+7}$$

$$2^{7-2n}$$

B) $\dfrac{(6^{-3})(2^{4})(9^{-2})}{(27^{-4})(4^{-2})}$ $= \dfrac{(2\cdot 3)^{-3}\cdot 2^{4}\cdot (3^{2})^{-2}}{(3^{3})^{-4}\cdot (2^{2})^{-2}}$

$$= \frac{2^{-3}\cdot 3^{-3}\cdot 2^{4}\cdot 3^{-4}}{3^{-12}\cdot 2^{-4}}$$

$$= \frac{2^{1}\cdot 3^{-7}}{3^{-12}\cdot 2^{-4}}$$

$$= 2^{1-(-4)}\cdot 3^{-7-(-12)} = 2^{5}\cdot 3^{5} = (2\cdot 3)^{5} = 6^{5}$$

Solve each of the following equations:

B) $5^{2x+3} = 5^{x-9}$

Since the bases are the same we can equate the exponents

$2x+3 = x-9$

$2x-x = -9-3$

$\boxed{x = -12}$

C) $8^{2x} = 2^{4x+1}$

→ get a common base

$(2^3)^{2x} = 2^{4x+1}$

$2^{6x} = 2^{4x+1}$

$6x = 4x+1$

$2x = 1$

$x = 1/2$

D) $3 = 48\left(\frac{1}{2}\right)^{\frac{x}{92}}$

$$\frac{3}{48} = \left(\frac{1}{2}\right)^{x/92}$$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^{x/92}$$

$$\frac{1}{2^4} = \left(\frac{1}{2}\right)^{x/92}$$

$$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{x/92}$$

$$4 = x/92$$

$$368 = x$$

E) $81^{2x} = 27^{4x-5}$

$$(3^4)^{2x} = (3^3)^{4x-5}$$

$$3^{8x} = 3^{12x-15}$$

$$8x = 12x-15$$

$$-4x = -15$$

$$x = \frac{15}{4}$$

F) $2(3^x) - 20 = 34$

$\quad +20 \quad +20$

$$\frac{2(3^x)}{2} = \frac{54}{2}$$

$$3^x = 27$$

$$3^x = 3^3$$

$$x = 3$$

G) $\sqrt[5]{8^{x-1}} = \sqrt[3]{16^x}$

$$8^{\frac{x-1}{5}} = 16^{x/3}$$

$$(2^3)^{\frac{x-1}{5}} = (2^4)^{x/3}$$

$$2^{\frac{3x-3}{5}} = 2^{4x/3}$$

$$\frac{3x-3}{5} = \frac{4x}{3}$$

$$(3x-3)3 = 5(4x)$$

$$9x - 9 = 20x$$

$$-9 = 11x$$

$$x = \frac{-9}{11}$$

H) $9^x = 3^x + 6$

$(3^2)^x = 3^x + 6$

$3^{2\cdot x} = 3^x + 6$

$3^{x\cdot 2} = 3^x + 6$

$(3^x)^2 = 3^x + 6$

$(3^x)^2 - 3^x - 6 = 0$ let $m = 3^x$

$m^2 - m - 6 = 0$

$(m-3)(m+2) = 0$

$(m-3)(m+2) = 0$

$m - 3 = 0$ $m + 2 = 0$

$m = 3$ $m = -2$

$3^x = 3$ $3^x = -2$

$3^x = 3^1$ NO SOLUTION

$x = 1$

HW: pg 364 #1-5, C2, sheet (#6,7)