20. If $\angle A$ and $\angle B$ are both in quadrant I, and $\sin A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$, evaluate each of the following.

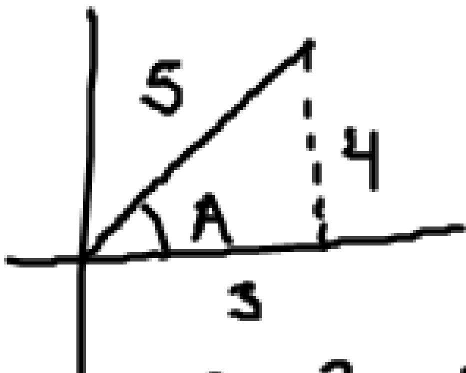
~~**a)** $\cos(A - B)$~~

b) $\sin(A + B)$ $= \sin A \cos B + \sin B \cos A$

$$\left(\frac{4}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$$

~~**c)** $\cos 2A$~~

~~**d)** $\sin 2A$~~

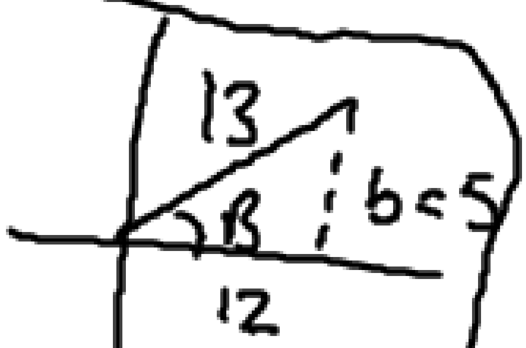


$a^2 + b^2 = c^2$

$a^2 + 4^2 = 5^2$

$a^2 = 9$

$a = 3$



$$\frac{48 + 15}{5(13)} = \frac{63}{65}$$

Ch 6.4 Solving trig Equations with Trig Identities Day 2

Example: Solve the following for the indicated interval in radians

A) $\cos x\cos\left(\frac{\pi}{5}\right)-\sin x\sin\left(\frac{\pi}{5}\right)=\frac{\sqrt{3}}{2}$ (all solutions)

let $x = A$

$\frac{\pi}{5} = B$

$\cos A\cos B - \sin A\sin B = \frac{\sqrt{3}}{2}$

$\cos(A+B) = \frac{\sqrt{3}}{2}$

$\cos\left(x+\frac{\pi}{5}\right) = \frac{\sqrt{3}}{2}$

$x+\frac{\pi}{5} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$x+\frac{\pi}{5}=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$x+\frac{\pi}{5}=\begin{cases}\frac{\pi}{6}\\ \frac{11\pi}{6}\end{cases}+2\pi k,\ k\in I$$

$$x+\frac{6\pi}{30}=\begin{cases}\frac{5\pi}{30}\\ \frac{55\pi}{30}\end{cases}+2\pi k,$$

$$x=\begin{cases}\frac{-\pi}{30}\\ \frac{49\pi}{30}\end{cases}+2\pi k,\ k\in I$$

B) $4\sin x\cos x=\sqrt{3}$ $\qquad$ $-2\pi \le x \le 2\pi$

$$2(2\sin x\cos x)=\sqrt{3}$$

$$\frac{2\sin(2x)}{2}=\frac{\sqrt{3}}{2}$$

QI+II

$$\sin(2x)=\frac{\sqrt{3}}{2}$$

$$2x=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$2x=\begin{cases}\frac{\pi}{3}\\ \frac{2\pi}{3}\end{cases}+2\pi k,\ k\in I$$

$$x=\begin{cases}\frac{\pi}{6}\\ \frac{2\pi}{6}\end{cases}+\pi k,\ k\in I$$

$\frac{6\pi}{6}$ $\quad$ $\frac{3\pi}{3}$

$$-\frac{12\pi}{6}\le x<\frac{12\pi}{6}$$

$\frac{6\pi}{3}$

$$x=\begin{cases}\frac{\pi}{6},\ \frac{7\pi}{6},\ -\frac{5\pi}{6},\ -\frac{11\pi}{6}\\ \frac{\pi}{3},\ \frac{4\pi}{3},\ -\frac{2\pi}{3},\ -\frac{5\pi}{3}\end{cases}$$

C) 2[illegible]1 $\qquad 0 \le x \le 2\pi$ $\quad \frac{12\pi}{6}$

$$\frac{2\cos 2x}{2} = \frac{1}{2}$$

$$\cos(2x) = \frac{1}{2}$$

$$2x = \cos^{-1}\left(\frac{1}{2}\right)$$

$$2x = \begin{cases} \frac{\pi}{3} \\ \frac{5\pi}{3} \end{cases} + 2\pi k,$$

$$x = \begin{cases} \frac{\pi}{6} & \frac{6\pi}{6} \\ \frac{5\pi}{6} \end{cases} + \pi k,\ k \in \mathbb{Z}$$

$$x = \begin{cases} \frac{\pi}{6},\ \frac{7\pi}{6}, \\ \frac{5\pi}{6},\ \frac{11\pi}{6} \end{cases}$$

12. If $\cos x = \frac{2}{3}$ and $\cos x = -\frac{1}{3}$ are the solutions for a trigonometric equation, what are the values of B and C if the equation is of the form $9\cos^2 x + B\cos x + C = 0$?

method 1 → fill in the values

$\cos x = \frac{2}{3}$

$9\left(\frac{2}{3}\right)^2 + B\left(\frac{2}{3}\right) + C = 0$

$\frac{36}{9} + \frac{2B}{3} + C = 0$

$4 + \frac{2B}{3} + C = 0$

$12 + 2B + 3C = 0$

$\boxed{2B + 3C = -12}$ ①

$\cos x = \frac{-1}{3}$

$9\left(\frac{-1}{3}\right)^2 + B\left(\frac{-1}{3}\right) + C = 0$

$\frac{9}{9} - \frac{B}{3} + C = 0$

$1 - \frac{B}{3} + C = 0$

$3 - B + 3C = 0$

$\boxed{-B + 3C = -3}$

mult by 2

substitution/eliminat

$$\begin{array}{r} 2B + 3C = -12 \\ + \underline{-2B + 6C = -6} \\ 9C = -18 \end{array}$$

$\boxed{C = -2}$

$B = 3C + 3$

$B = 3(-2) + 3$

$\boxed{B = -3}$

Method 2 - work Backwards

$\cos x = \frac{2}{3}$ $\qquad\qquad$ $\cos x = -\frac{1}{3}$

$3\cos x = 2$ $\qquad\qquad$ $3\cos x = -1$

$3\cos x - 2 = 0$ $\qquad\qquad$ $3\cos x + 1 = 0$

$$(3\cos x - 2)(3\cos x + 1) = 0$$

$$9\cos^2 x + 3\cos x - 6\cos x - 2 = 0$$

$$9\cos^2 x - 3\cos x - 2 = 0$$

$B = -3 \qquad c = -2$

Pg 321

Extend

17. Solve $4 \sin^2 x = 3 \tan^2 x - 1$ algebraically. Give the general solution expressed in radians.

*check NPV

1) used ident $\tan\theta = \frac{\sin\theta}{\cos\theta}$

2) mult. each term by $\cos^2 x$

3) use ident $\sin^2\theta + \cos^2\theta = 1$ to get only sine terms

$$4\sin^2 x = 3\left(\frac{\sin^2 x}{\cos^2 x}\right) - 1$$

$$4\sin^2 x\cos^2 x = 3\sin^2 x - \cos^2 x$$

$$4\sin^2 x(1-\sin^2 x) = 3\sin^2 x - (1-\sin^2 x)$$

$$4\sin^2 x - 4\sin^4 x = 3\sin^2 x - 1 + \sin^2 x$$

$$0 = 4\sin^4 x - 1$$

HW:pg 321# 6,14,16,18,C2

$0 = 4\sin^4 x - 1$ let $a = \sin x$

$0 = 4a^4 - 1$

$0 = (2a^2 - 1)(2a^2 + 1)$

$2a^2 - 1 = 0$ $2a^2 + 1 = 0$

$a^2 = \frac{1}{2}$ $a^2 = -\frac{1}{2}$

$\sin^2 x = \frac{1}{2}$ $\sin^2 x = -\frac{1}{2}$ ← No SOL'N

$\sin x = \pm\sqrt{\frac{1}{2}}$

$\sin x = \frac{\sqrt{2}}{2}$ $\sin x = -\frac{\sqrt{2}}{2}$

$x = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right. + 2\pi k, k \in I$

NPV:

$\cos x \neq 0$

$$x \neq \begin{cases} \frac{\pi}{2} \\ \frac{3\pi}{2} \end{cases} + 2\pi k,$$

we're ok.....

HW:pg 321# 6,14,16,18,C2

Pg 321

Extend

17. Solve $4 \sin^2 x = 3 \tan^2 x - 1$ algebraically. Give the general solution expressed in radians.

1) use $\tan\theta = \frac{\sin\theta}{\cos\theta}$

★ NPV

$\cos x \neq 0$

2) mult each term by $\cos^2 x$

$$4\sin^2 x = 3\frac{\sin^2 x}{\cos^2 x} - 1$$

$$4\sin^2 x\cos^2 x = 3\sin^2 x - \cos^2 x$$

$$4\sin^2 x\cos^2 x - 3\sin^2 x + \cos^2 x = 0$$

use $\sin^2\theta + \cos^2\theta = 1$

HW:pg 321# 6,14,16,18,C2

$$4\sin^2 x\cos^2 x - 3\sin^2 x + \cos^2 x = 0$$

use $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$$4(1-\cos^2 x)\cos^2 x - 3(1-\cos^2 x) + \cos^2 x = 0$$

$$4\cos^2 x - 4\cos^4 x - 3 + 3\cos^2 x + \cos^2 x = 0$$

$$-4\cos^4 x + 8\cos^2 x - 3 = 0$$

$$4\cos^4 x - 8\cos^2 x + 3 = 0$$

let $A = \cos x$

$$4A^4 - 8A^2 + 3 = 0$$

Quartic as a Quadratic

$4A^4 - 8A^2 + 3 = 0$ let $B = A^2$

$4B^2 - 8B + 3 = 0$

12

$\_ \times \_ = 12$

$\_ + \_ = -8$

$\underbrace{4B^2 - 6B} - \underbrace{2B + 3} = 0$

$2B(2B-3) - 1(2B-3) = 0$

$(2B-1)(2B-3) = 0$

$B = \frac{1}{2}$ $B = \frac{3}{2}$

$A^2 = \frac{1}{2}$ $A^2 = \frac{3}{2}$

$\cos^2 x = \frac{1}{2}$ $\cos^2 x = \frac{3}{2}$

$\cos^2 x = \frac{1}{2}$ $\quad$ $\cos^2 x = \frac{3}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}}$ $\quad$ $\cos x = \pm\sqrt{\frac{3}{2}}$

$\cos x = \pm\frac{\sqrt{2}}{2}$ $\quad$ NO SOL'N

$\cos x = -\frac{\sqrt{2}}{2}$ $\quad$ $\cos x = +\frac{\sqrt{2}}{2}$

$x = \begin{cases} \frac{3\pi}{4} \\ \frac{5\pi}{4} \end{cases} + 2\pi k$ $\quad$ $x = \begin{cases} \frac{\pi}{4} \\ \frac{7\pi}{4} \end{cases} + 2\pi k$ $\quad$ $k \in I$

NPV

$\cos x \neq 0$

$x \neq \frac{\pi}{2}, \frac{3\pi}{2}$

HW:pg 321# 6,14,16,18,C2