$$\sin(2x) = \frac{\sqrt{2}}{2}$$

let $a = 2x$

$$\sin a = \frac{\sqrt{2}}{2}$$

$$\sin(2x) - \cos x = 0$$

$$2\sin x\cos x - \cos x = 0$$

4. Solve $4 \sin^2 x = 1$ algebraically over the domain $-180° \le x < 180°$.

Q I + II

Q III + IV

$\sin^2 x = \frac{1}{4}$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{4}}$

$\sin x = \pm \frac{1}{2}$

$\sin x = \frac{1}{2}$

$x = \begin{cases} 30° \\ 150° \end{cases} + 360°k,$

$\sin x = -\frac{1}{2}$

$x = \begin{cases} 210° \\ 330° \end{cases} + 360°k$

$x = \begin{cases} 30°, 150° \\ -150°, -30° \end{cases}$

3. Rewrite each equation in terms of sine only. Then, solve algebraically for $0 \leq x < 2\pi$.

a) $\cos 2x - 3 \sin x = 2$

b) $2 \cos^2 x - 3 \sin x - 3 = 0$

c) $3 \csc x - \sin x = 2$

d) $\tan^2 x + 2 = 0$

NPV: $\sin x \neq 0$

$x \neq 0, \pi,$

$3 \csc x - \sin x = 2$

$\frac{3}{\sin x} - \sin x = 2$

mult each term $\sin x$

$3 - \sin^2 x = 2 \sin x$

$0 = \sin^2 x + 2 \sin x - 3$

$0 = (\sin x + 3)(\sin x - 1)$

$\sin x = -3$

No soln

$\sin x = 1$

$x = \left\{ \frac{\pi}{2} \right.$

Proving Trig Identities – Day 3

Examples:

identity

$\frac{abc}{bcd}$

$\frac{a+b}{ab}$

1. Prove: $1 - \tan x \tan y = \dfrac{\cos(x+y)}{\cos x \cos y}$

$$\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}$$

$$\frac{\cancel{\cos x}\,\cancel{\cos y}}{\cancel{\cos x}\,\cancel{\cos y}} - \frac{\sin x \sin y}{\cos x \cos y}$$

$$1 - \tan x \tan y$$

2. Prove: $\dfrac{\sin 3x - \sin x}{\cos 3x + \cos x} = \tan x$

LHS:

$$\frac{\sin(2x+x) - \sin x}{\cos(2x+x) + \cos x}$$

$$\frac{(\sin 2x)\cos x + \sin x \cos 2x - \sin x}{\cos 2x \cos x - \sin 2x \sin x + \cos x}$$

$$\frac{(2\sin x \cos x)\cos x + \sin x(1-2\sin^2 x) - \sin x}{(2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x + \cos x}$$

$$\frac{(2\sin x\cos x)\cos x + \sin x(1-2\sin^2 x) - \sin x}{(2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x + \cos x}$$

$$\frac{2\sin x\cos^2 x \cancel{+\sin x} - 2\sin^3 x \cancel{-\sin x}}{2\cos^3 x \cancel{-\cos x} - 2\sin^2 x\cos x \cancel{+\cos x}}$$

$$\frac{2\sin x\cos^2 x - 2\sin^3 x}{2\cos^3 x - 2\sin^2 x\cos x}$$

$$\frac{\cancel{2}\sin x\,\cancel{(\cos^2 x - \sin^2 x)}}{\cancel{2}\cos x\,\cancel{(\cos^2 x - \sin^2 x)}}$$

$$\frac{\sin x}{\cos x} = \tan x = RHS$$

we are awesome :)