MY WEBSITE

Lmurphymath.weebly.com

Reminder Texts/Emails

Text @chspc12 to (902)708-0495

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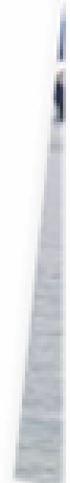
3.3

The Factor Theorem



Focus on...

- factoring polynomials
- explaining the relationship between the linear factors of a polynomial expression and the zeros of the corresponding function
- modelling and solving problems involving polynomial functions

Remember $P(x)$ divided by $(x-a)$ the remainder is $P(a)$

$$P(3) = x^2 + 6x + 8$$
$$= 3^2 + 6(3) + 8$$
$$= 9 + 18 + 8$$
$$= 35$$

Remainder of 0 means its a perfect factor

The Factor theorem states that a polynomial $P(x)$, has a factor $x-a$ if and only if (iff) $P(a)=0$ → remainder is 0

Example: $P(x)=x^5-x^4+x^3-2x+1$ has a factor of $x-1$

$$P(1) = (1)^5-(1)^4+(1)^3-2(1)+1$$

Factor

$x-1$

$$= 1-1+1-2+1$$

Root

$x=1$

$$= 0+1-2+1$$

$$= 0$$

Integral Zero Theorem- if $x - a$ is a factor of a polynomial $P(x)$, then a is a factor of the constant term

Factors of 12

12

1 12

2 6

3 4

$P(x) = x^2 + 8x + 12$

$(x+2)(x+6)$

$x+2$ is a factor so $x = -2$ is a root

$P(-2) = (-2)^2 + 8(-2) + 12$

$= 4 - 16 + 12$

$= 0$

Example: What are the possible zeros of the following polynomial? $P(n) = n^3 - 3n^2 - 10n + 24$

all the Factors of 24

$a = \pm 1 \pm 2 \pm 3 \pm 4 \pm 6 \pm 8 \pm 12 \pm 24$

The Integral Zero Theorem is a specialized case of the Rational Root Theorem

Factor

$(ax-b)$ is a factor of $P(x)$ if and only if (iff) $P\left(\frac{b}{a}\right)=0$.

$ax-b=0$

$ax=b$

$x=b/a$ (root)

ex.

$(3x-2)(x+3)(2x-5)$

$6x^3 \cdots\cdots +30$

Recall:

$$P(x)=a_nx^n+a_{n-1}x^{n-1}+\ldots+a_3x^3+a_2x^2+a_1x^1+a_0$$

b is the factors of a_0 and a is the factors of a_n

constant term

leading coefficient

Example: write $P(x) = x^3 - 7x^2 - 4x + 28$ in factored form

$(ax - b)$

Step 1- find the factors of the coefficient of the leading term and the constant term

b = comes from 28 = $\pm 1 \pm 2 \pm 4 \pm 7 \pm 14 \pm 28$

a = comes from 1 = ± 1

$b/a = \pm 1 \pm 2 \pm 4 \pm 7 \pm 14 \pm 28$

Step 2- calculate $P\left(\frac{b}{a}\right)$. If it equals zero, it is a rational root of $P(x)$.

$$P(2) = 2^3 - 7(2^2) - 4(2) + 28$$
$$= 8 - 28 - 8 + 28$$
$$= 0$$

x=2 (root)

x-2 (factor)

Step 3- Using a root, create the factor $(ax - b)$ and factor polynomial.

$$x-2\,\overline{)\,x^3-7x^2-4x+28}$$

$$\begin{array}{r|rrrr} 2 & 1 & -7 & -4 & 28 \\ & \downarrow & 2 & -10 & -28 \\ \hline & 1 & -5 & -14 & \boxed{0} \end{array}$$

Step 4- Repeat until fully factored ☺

$(x-2)(x^2-5x-14)$

$(x-2)(x-7)(x+2)$

x ints: $(2,0)$

$(7,0)$

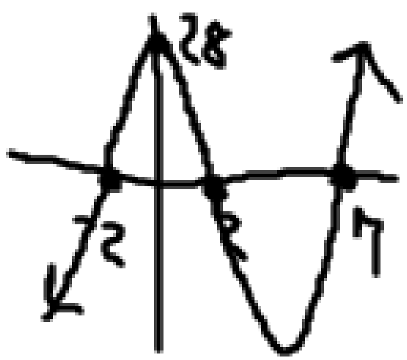
$(-2,0)$

y int $(0,28)$

end behavior: + cubic

start [illegible]

end high



Example: factor $g(x) = 4x^3 - 12x^2 + 5x + 6$

(1) constant = 6 → Factors of 6: $\pm 1, \pm 2, \pm 3, \pm 6$

(4) LC = 4 → factors of 4: $\pm 1, \pm 2, \pm 4$

$\frac{6}{4} = \pm 1 \quad \pm 2 \quad \pm 3 \quad \pm 6 \quad \pm 1/2 \quad \pm 3/2 \quad \pm 1/4 \quad \pm 3/4$

$$g(2) = 4(2)^3 - 12(2)^2 + 5(2) + 6$$
$$= 32 - 48 + 10 + 6$$
$$= 0$$

x = 2 is a root

(x − 2) is a factor

$P(x) = 4x^3 - 12x^2 + 5x + 6$

$(x-2)$ Factor

$$\begin{array}{r|rrrr} 2 & 4 & -12 & 5 & 6 \\ & \downarrow & 8 & -8 & -6 \\ \hline & 4 & -4 & -3 & \boxed{0} \end{array}$$

$(x-2)(4x^2-4x-3)$

$(x-2)(2x+1)(2x-3)$

x ints $(2,0)$

$(^{-1}/_{2}, 0)$

$(^{3}/_{2}, 0)$

y int $(0,6)$

end behavior

Starts low ends high

Example: find the degree 4 polynomial with x int of (1,0) (-1,0) (-1/2, 0) and (-2,0) and a y int of (0, -2)

$P(x) = a(x-1)(x+1)(2x+1)(x+2)$

Check: $-1 \cdot 1 \cdot 1 \cdot 2 = -2 \checkmark$

y int

So $a = 1$

$P(x) = (x-1)(x+1)(2x+1)(x+2)$

HW: pg 133 #1-7