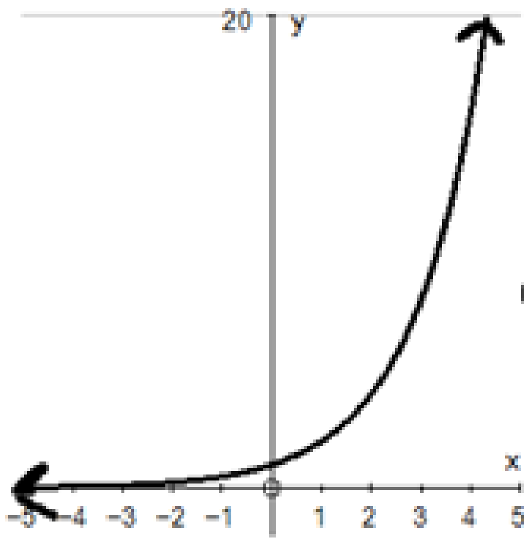
Chapter 7.1 & 7.2 – Exponential Functions

- The simplest exponential function is $y = c^x$ where $c>0$, $c \in R$
- Two types of graphs produced:

c is a constant
x is a variable

Increasing (growth) curve



$y=c^x$

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y>0, y \in R\}$

HA: $y=0$

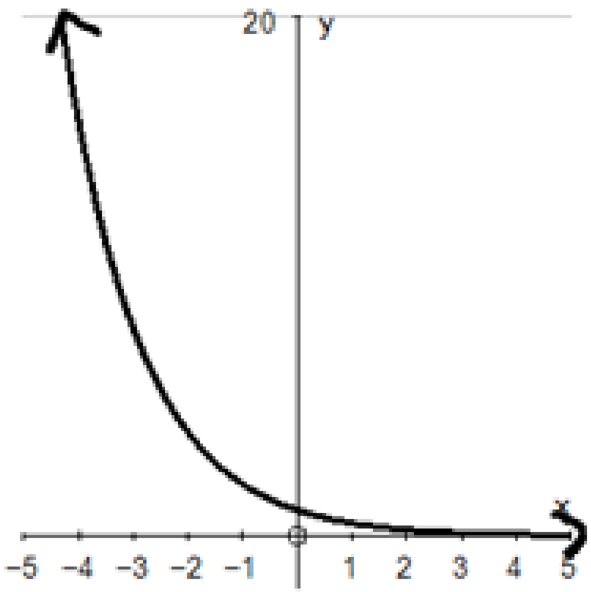
x-int: NONE

y-int: $(0,1)$

$y=c^0$
$y=1$

$c>1$
increasing growth
(growth)

decreasing (decay) curve



Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y > 0, y \in \mathbb{R}\}$

HA: $y = 0$

x-int: None

y-int: $(0, 1)$

$$y = c^x$$

$$0 < c < 1$$

not equal to → $\underbrace{y = 1^x}_{\text{not exponential (linear)}}$

$$y = c^x$$

$c > 0$

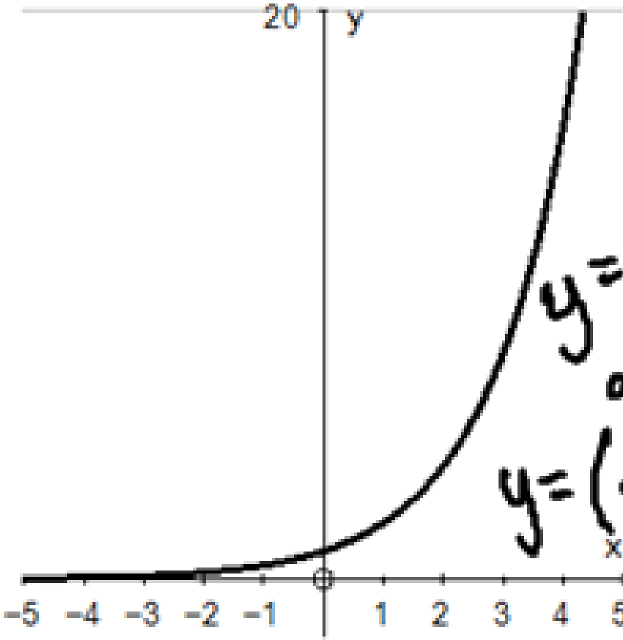
$c \neq 1$

if $c > 1$ = growth

if $0 < c < 1$ = decay

Transformations: $y = a(c)^{b(x-h)} + k$

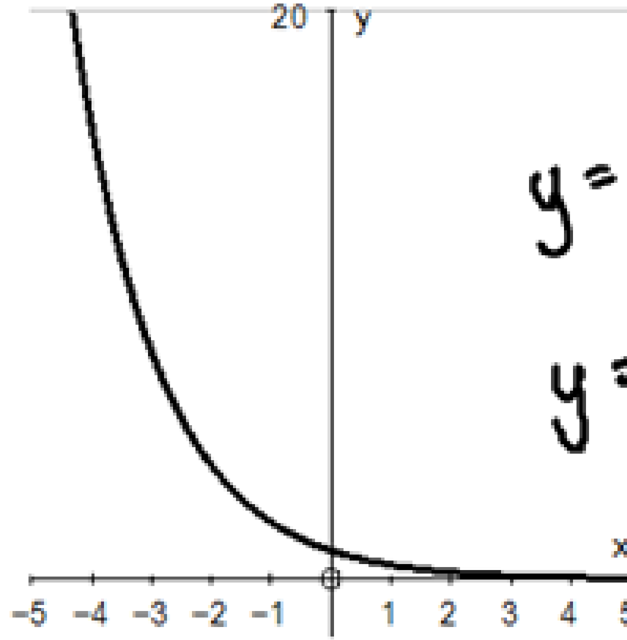
Reflections in the y- axis



$y = c^x$

or

$y = \left(\frac{1}{c}\right)^{-x}$



$y = \left(\frac{1}{c}\right)^x$

or

$y = c^{-x}$

$y = c^x \longrightarrow y = c^{-x}$

changes from growth to decay

OR

$y = \left(\frac{1}{c}\right)^{-x} \longleftarrow y = \left(\frac{1}{c}\right)^x$

changes from decay to growth

$y = c^x$ (growth) $c > 1$

$y = c^{-x}$ reflection in y axis

$y = c^{-1 \cdot x}$

$y = (c^{-1})^x$

$y = \left(\frac{1}{c}\right)^x$

$y = \left(\frac{1}{c}\right)^x$ (decay) $0 < c < 1$

$y = \left(\frac{1}{c}\right)^{-x}$ reflection in y axis

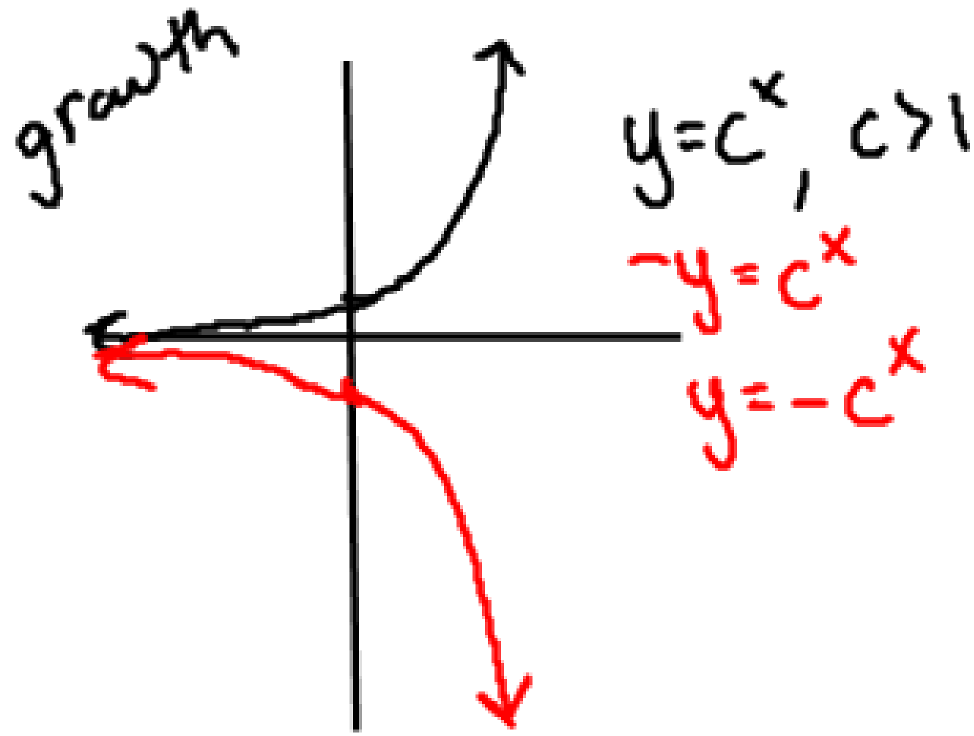
$y = \left(\frac{1}{c}\right)^{-1 \cdot x}$

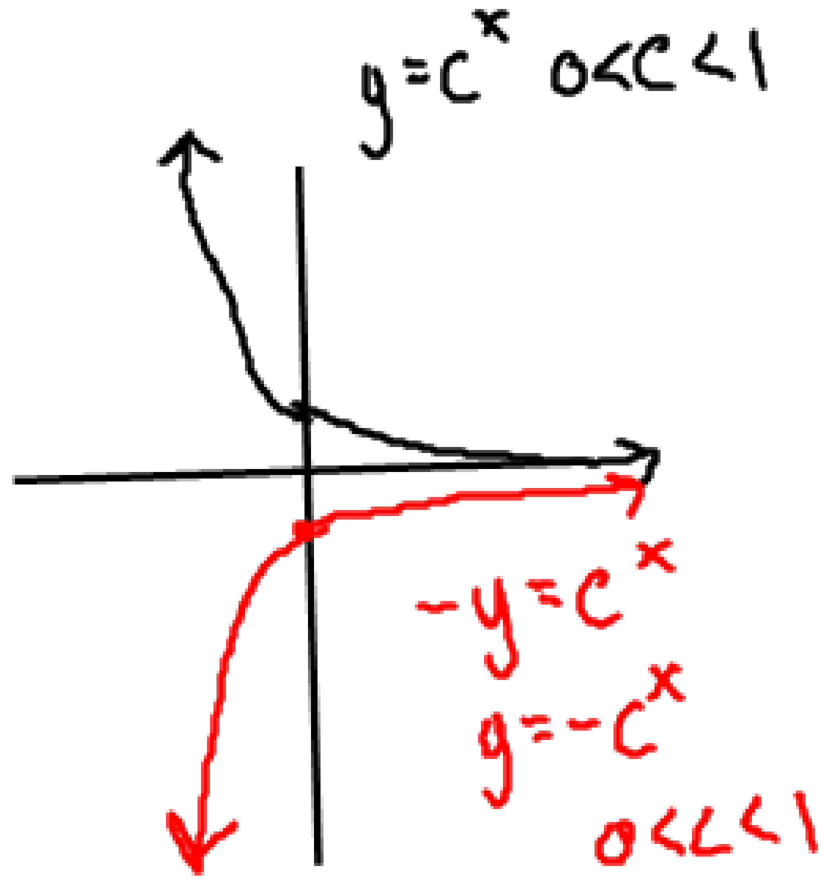
same thing

$y = \left(\left(\frac{1}{c}\right)^{-1}\right)^x$

$y = (c^1)^x \rightarrow y = c^x$

Reflections in the x- axis





changed to decay
y int changes to (0, -1)
range changes to $\{y \mid y<0, y \in R\}$

Changed from decay to growth
y int (0, -1)
Range: $\{y \mid y<0, y \in R\}$

To Decide if its Growth or Decay

① $c > 1$ = growth or $0 < c < 1$ = decay

② $-x$ switches it

③ $-y$ switches it (or $-a$)

ex. $y = (1/2)^{-x} \Rightarrow$ growth

$y = -(3)^{-x} \Rightarrow$ growth

$y = -(1/3)^{x} \Rightarrow$ growth

$y = -3^x$

* $-a$ is okay (reflection in x axis)
  $-c$ is not okay (not exponential) $\cancel{y = (-3)^x}$

Mapping Rule:

the transformed exponential $y = \underbrace{a}_{VS}(c)^{\overbrace{b}^{1/HS}(x \overbrace{-h}^{HT})} + \underbrace{k}_{VT}$ would be

$$(x, y) \rightarrow \left(\underbrace{(-)}_{Ry}\frac{1}{b}x + h, \underbrace{(-)}_{Rx}ay + k\right)$$

Examples: graph the following $y = 2(3)^{-x-3} + 1$ → needs to be factored $-1(x+3)$

List transformations: VS = 2 HS = 1 Ry
VT = +1 HT = −3

HA: $\boxed{y = VT}$ $y = 1$

y-int: $x = 0$
$y = 2(3)^{-3} + 1 = 2\left(\frac{1}{27}\right) + \frac{27}{27}$
$\left(0, \frac{29}{27}\right)$

x-int:
NONE

range:

$\{y \mid y > 1, y \in R\}$

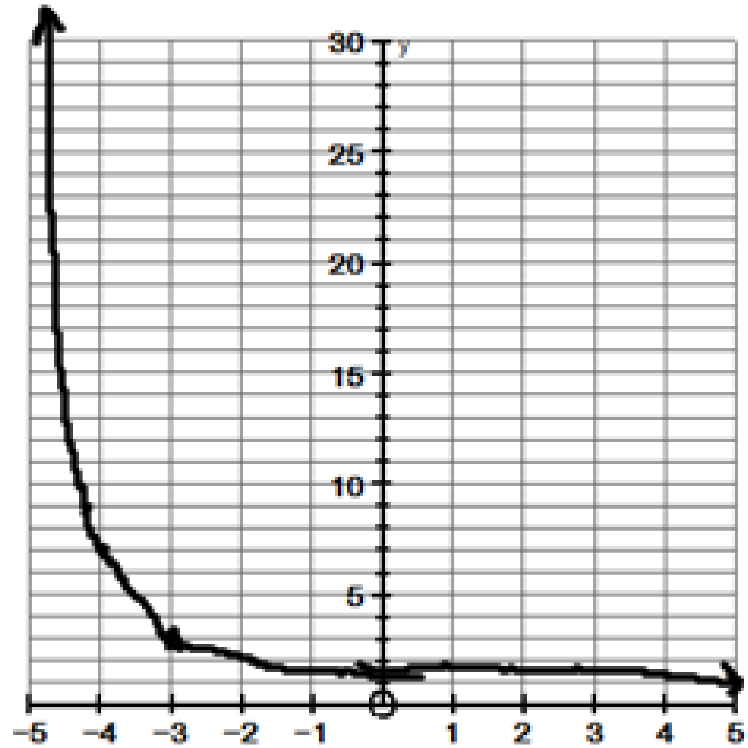
Mapping Rule:

$(x, y) \rightarrow (-x-3, 2y+1)$

Table of values…

x	$y = 3^x$
−3	1/27
−2	1/9
−1	1/3
0	1
1	3
2	9
3	27

$-x-3$	$2y+1$
0	29/27
−1	11/9
−2	5/3
−3	3
−4	7
−5	19



30
25
20
15
10
5
y
–5
–4
–3
–2
–1
1
2
3
4
5

Apply

7. Describe the transformations that must be applied to the graph of each exponential function $f(x)$ to obtain the transformed function. Write each transformed function in the form $y = a(c)^{b(x-h)} + k$.

a) $f(x) = \left(\frac{1}{2}\right)^x$, $y = f(x - 2) + 1$

f

apply the transformations to this equation

HT 2

VT 1

$$\boxed{y = \left(\frac{1}{2}\right)^{x-2} + 1}$$

d) $f(x) = 4^x$, $y = 2f\left(-\frac{1}{3}(x-1)\right) - 5$

VS 2

Ry

HS 3

HT 1

VT −5

$$\boxed{y = 2(4)^{-\frac{1}{3}(x-1)} - 5}$$

TRANSFORMATIONAL FORM

$\frac{1}{2}(y+5) = 4^{-1/3(x-1)}$

$\frac{1}{VS}(y - VT) = C^{\frac{1}{HS}(x-HT)}$

note: focal point is y intercept

HW: pg 355 # 6,7 & sheet