6.4

Solving Trigonometric Equations Using Identities

Focus on...

- solving trigonometric equations algebraically using known identities
- determining exact solutions for trigonometric equations where possible
- determining the general solution for trigonometric equations
- identifying and correcting errors in a solution for a trigonometric equation

Examples:

Solve each equation algebraically over the domain indicated.

(a) $\sin 2x - \cos x = 0$ $\qquad 0 \le x \le 2\pi$

NO COMMON FACTOR

$\sin 2A = 2\sin A\cos A$

$2\sin x\cos x - \cos x = 0$

COMMON FACTOR $\cos x$

$\cos x(2\sin x - 1) = 0$

SET EACH FACTOR EQUAL TO ZERO AND SOLVE FOR x

$\cos x = 0$

$x = \begin{cases} \pi/2 \\ 3\pi/2 \end{cases} + 2\pi n, \quad n \in I$

$2\sin x - 1 = 0$

$\sin x = 1/2$

$x = \begin{cases} \pi/6 \\ 5\pi/6 \end{cases} + 2\pi n, \quad n \in I$

$$\boxed{x = \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}}$$

(b) $2\cos x + (1 - \sin^2 x) = 3 \qquad 0 \le x \le 2\pi$

$= \cos^2 x \quad \leftarrow \quad \sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$2\cos x + \cos^2 x = 3$

$\cos^2 x + 2\cos x - 3 = 0$

$(\cos x + 3)(\cos x - 1) = 0$

$\cos x + 3 = 0 \qquad \cos x - 1 = 0$

$\cos x = -3 \qquad \cos x = 1$

NO SOL'N $\qquad x = \{0 + 2\pi k, \ k \in I$

$$\boxed{x = \{0, 2\pi\}}$$

(c) $\sin^2 x = \frac{1}{2}\tan x \cos x \qquad 0^\circ \le x \le 360^\circ$

USE IDENTITY $\tan x = \frac{\sin x}{\cos x}$

$$\sin^2 x = \frac{1}{2}\left(\frac{\sin x}{\cancel{\cos x}}\right)\left(\cancel{\cos x}\right)$$

WE HAVE INTRODUCED A FRACTION, MUST LOOK FOR NON-PERMISSIBLE VALUES

$\cos x \neq 0$

$x \neq \begin{cases} 90^\circ \\ 270^\circ \end{cases} + 360^\circ n,\ n \in I$

$$\sin^2 x = \frac{1}{2}\sin x$$

$$\sin^2 x - \frac{1}{2}\sin x = 0$$

$$\sin x\left(\sin x - \frac{1}{2}\right) = 0$$

CHECK THESE AGAINST SOLUTIONS

ALL ARE OKAY!

$\sin x = 0 \qquad \sin x = \frac{1}{2}$

$x = \begin{cases} 0^\circ \\ 180^\circ \end{cases} + 360^\circ n,\ n \in I \qquad x = \begin{cases} 30^\circ \\ 150^\circ \end{cases} + 360^\circ n,\ n \in I$

$$\boxed{x = \{0^\circ, 30^\circ, 150^\circ, 180^\circ, 360^\circ\}}$$

(d) $\cos 2x = \cos x$ all solutions (radians)

USE $\cos 2x = 2\cos^2 x - 1$

$$2\cos^2 x - 1 = \cos x$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1)$$

$2\cos x + 1 = 0$

$\cos x = -1/2$

$$x = \begin{cases} 2\pi/3 \\ 4\pi/3 \end{cases} + 2\pi n,\ n \in I$$

$\cos x - 1 = 0$

$\cos x = 1$

$$x = \{0 + 2\pi n,\ n \in I$$

$$\boxed{x = \begin{cases} 0 \\ 2\pi/3 \\ 4\pi/3 \end{cases} + 2\pi n,\ n \in I}$$

(e) $3\cos x + 2 = 5\sec x$ all solutions (radians)

USE $\sec x = \dfrac{1}{\cos x}$

N.P.V.

$\cos x \neq 0$

$x \neq \begin{cases} \pi/2 \\ 3\pi/2 \end{cases} + 2\pi k,$ $k \in I$

(DOESN'T AFFECT SOLUTION)

$$3\cos x + 2 = \frac{5}{\cos x}$$

MULTIPLY BY $\cos x$

$$3\cos^2 x + 2\cos x = 5$$

$$3\cos^2 x + 2\cos x - 5 = 0$$

$$(3\cos x + 5)(\cos x - 1) = 0$$

$3\cos x + 5 = 0$

$\cos x = -5/3$

NO SOL'N

$\cos x - 1 = 0$

$\cos x = 1$

$\boxed{x = 0 + 2\pi k, k \in I}$

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