4) a) $\dfrac{\sec x - \cos x}{\tan x} = \dfrac{\dfrac{1}{\cos x} - \cos x}{\dfrac{\sin x}{\cos x}}$

b) $\dfrac{\dfrac{1}{\cos x} - \dfrac{\cos^2 x}{\cos x}}{\dfrac{\sin x}{\cos x}} = \dfrac{1-\cos^2 x}{\cos x} \div \dfrac{\sin x}{\cos x} = \dfrac{\sin^{\cancel{2}} x}{\cancel{\cos x}} \cdot \dfrac{\cancel{\cos x}}{\cancel{\sin x}}$

$= \sin x$ ✓

pg 306 15b)

$$\frac{\csc^2 x - 2}{\csc^2 x} = \frac{\csc^2 x}{\csc^2 x} - \frac{2}{\csc^2 x}$$

$$= 1 - \frac{2}{\csc^2 x}$$

$$= 1 - 2\left(\frac{1}{\csc^2 x}\right)$$

$$= 1 - 2\sin^2 x$$

$$= \cos 2x \checkmark$$

5b) $\cos(2x)\cos x + \sin(2x)\sin x$

$\cos(2x)\cos x + (2\sin x\cos x)(\sin x)$

$\cos(2x)\cos x + 2\sin^2 x\cos x$

$(1 - 2\sin^2 x)(\cos x) + 2\sin^2 x\cos x$

$\cos x - 2\sin^2 x\cos x + 2\sin^2 x\cos x$

$\cos x$ ✓

18) $\sin(5x)\cos x + \cos(5x)\sin x = 2\sin(kx)\cos(kx)$

$A = 5x \quad B = x$

$\sin A\cos B + \cos A\sin B$

$\sin(A+B)$

$\sin(5x+x)$

* $\sin(6x)$

$\sin(2(3x))$ Double angle identity.

let $\theta = 3x$

$\sin(2\theta)$

$2\sin\theta\cos\theta \rightarrow 2\sin(3x)\cos(3x)$

$k = 3$ ✓

Ch 6.3 Proving Identities DAY 2

Prove that $\dfrac{\sin 2x}{\cos 2x + 1} = \tan x$ is an identity for all permissible values of x.

*sometimes writing out square terms as a product will help you see what to simplify

$\dfrac{\sin 2x}{\cos 2x + 1}$	$\tan x$
$\dfrac{2\sin x\cos x}{2\cos^2 x - 1 + 1}$	
$\dfrac{2\sin x\cos x}{2\cos^2 x}$	
$\dfrac{\cancel{2}\sin x\,\cancel{\cos x}}{\cancel{2}\cos x\,\cancel{\cos x}}$	
$\sin x/\cos x = \tan x$	$\tan x$ ✓

*looking for one term so if I can cancel the 1 on the bottom then I won't have two terms on bottom

Prove: $\dfrac{2\tan x}{1+\tan^2 x} = \sin 2x$

$\dfrac{2\tan x}{\sec^2 x}$

$2\dfrac{\sin x}{\cos x} \div \dfrac{1}{\cos^2 x}$

$\dfrac{2\sin x}{\cos x} \cdot \dfrac{\cos^2 x}{1}$

$\dfrac{2\sin x \cos x \cancel{\cos x}}{\cancel{\cos x}}$

$2\sin x \cos x$

$\sin(2x) \rightarrow$

$\sin(2x)$ ✓

* whenever you see a squared term think pythagorean identities.

+ on one side
no + on other side } try to get rid of

Prove that $\dfrac{\sin 2x - \cos x}{4\sin^2 x - 1} = \dfrac{\sin^2 x \cos x + \cos^3 x}{2\sin x + 1}$ is an identity for all permissible values of x.

diff of squares

$(a^2 - b^2)$

$(a+b)(a-b)$

$$\frac{2\sin x\cos x - \cos x}{(2\sin x + 1)(2\sin x - 1)}$$

$$\frac{\cos x\,\cancel{(2\sin x - 1)}}{(2\sin x + 1)\cancel{(2\sin x - 1)}}$$

$$\frac{\cos x \cdot 1}{2\sin x + 1}$$

$$\frac{\cos x\,(\sin^2 x + \cos^2 x)}{2\sin x + 1}$$

$$\frac{\sin^2 x\cos x + \cos^3 x}{2\sin x + 1} \rightarrow \frac{\sin^2 x\cos x + \cos^3 x}{2\sin x + 1} \checkmark$$

$$\sec 2a + \tan 2a = \frac{\cos a + \sin a}{\cos a - \sin a}$$

$$\frac{1}{\cos(2a)} + \frac{\sin(2a)}{\cos(2a)}$$

$$\frac{1+\sin(2a)}{\cos(2a)}$$

$$\frac{1+2\sin a\cos a}{\cos^2 a - \sin^2 a} \rightarrow \text{diff of squares}$$

$$(a^2-b^2)$$

$$(a+b)(a-b)$$

$$\frac{1+2\sin a\cos a}{(\cos a+\sin a)(\cos a-\sin a)}$$

$$\frac{1+2\sin a\cos a}{(\cos a+\sin a)(\cos a-\sin a)}$$

$$\frac{\cos a+\sin a}{\cos a-\sin a}$$

$$\frac{\sin^2 a+\cos^2 a+2\sin a\cos a}{(\cos a+\sin a)(\cos a-\sin a)}$$

→ rearrange to look familiar

$(a+b)^2 = a^2+2ab+b^2$

$$\frac{\sin^2 a+2\sin a\cos a+\cos^2 a}{(\cos a+\sin a)(\cos a-\sin a)}$$

*fancy version of 1

$\sin^2 a+\cos^2 a$

$$\frac{(\sin a+\cos a)\cancel{(\sin a+\cos a)}}{\cancel{(\cos a+\sin a)}(\cos a-\sin a)}$$

$$\frac{\cos a+\sin a}{\cos a-\sin a} \Rightarrow \frac{\cos a+\sin a}{\cos a-\sin a}$$ ✓

hw: worksheet proving trig identities