$$\frac{\csc^2 x - \cot^2 x}{\cos x}$$

$$\frac{\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}}{\cos x} = \frac{1 - \cos^2 x}{\sin^2 x} \div \cos x = \frac{\sin^2 x}{\sin^2 x} \div \cos x$$

$$= \frac{1}{\cos x}$$

$$= \sec x$$

b) $$\frac{\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}}{\cos x}$$

$\sin^2 x \neq 0 \longrightarrow \sin x \neq 0 \qquad x \neq 0, \pi \pm 2\pi k, k \in I$

$\cos x \neq 0 \longrightarrow x \neq \frac{\pi}{2}, \frac{3\pi}{2} \pm 2\pi k, k \in I$

$x \neq \left\{ 0, \frac{\pi}{2}, \frac{3\pi}{2}, \pi \pm 2\pi k, k \in I \right\}$

n)
$$\frac{2-\cos^2 x}{\sin x} = m + \sin x$$

$$\frac{2-\cos^2 x}{\sin x} - \sin x = m$$

$$\frac{2-\cos^2 x}{\sin x} - \frac{\sin^2 x}{\sin x} = m$$

$$\frac{2-\cos^2 x - \sin^2 x}{\sin x} = m$$

$$\frac{-1(-2+\cos^2 x + \sin^2 x)}{\sin x} = m$$

$$\frac{-1(-2+1)}{\sin x} = m$$

$$\frac{2-1}{\sin x} = m$$

$$\frac{1}{\sin x} = m$$

$$\boxed{\csc x = m}$$

6.2

Sum, Difference, and Double-Angle Identities

Focus on...

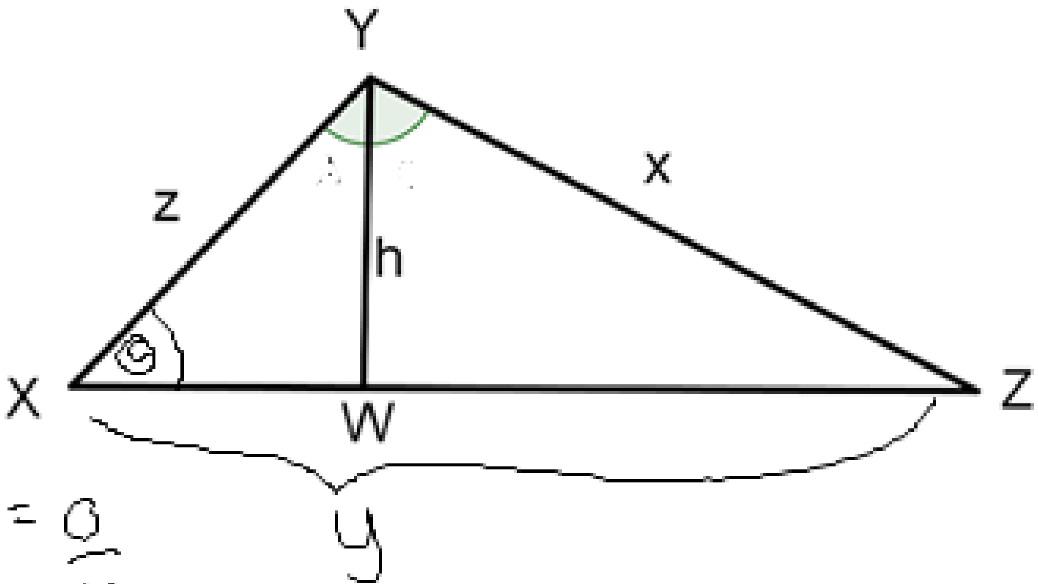
- applying sum, difference, and double-angle identities to verify the equivalence of trigonometric expressions
- verifying a trigonometric identity numerically and graphically using technology

Sum & Difference Identities:

Sine:

Area of a triangle:

$$A = \frac{b \times h}{2}$$

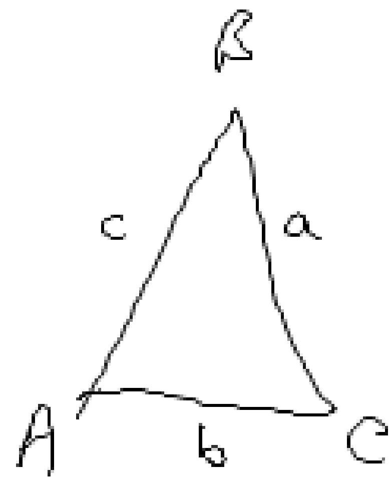


$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{h}{z}$

$z\sin\theta = h$

$A = \frac{y \cdot z\sin X}{2}$



Area formula uses 2 adjacent sides and enclosed angle

$$\boxed{A = \frac{ac\sin B}{2} \qquad A = \frac{ab\sin C}{2} \qquad A = \frac{bc\sin A}{2}}$$

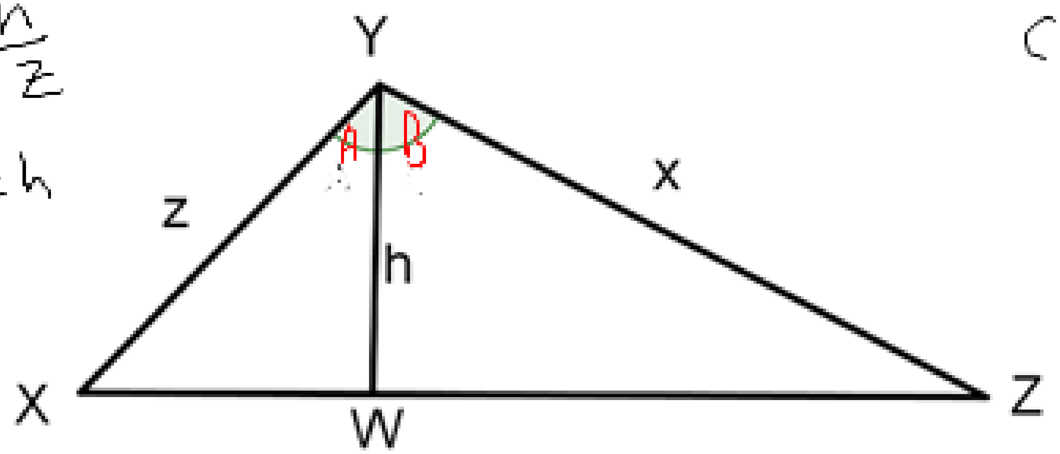
Sum & Difference Identities:

$\cos\theta = \frac{A}{H}$

Sine:

$\cos A = \frac{h}{z}$

$z\cos A = h$



$\cos B = \frac{h}{x}$

$h = \underline{x\cos B}$

$A\ \Delta_{xyz} = A_{\Delta xyw} + A_{\Delta wyz}$

$\frac{1}{2}x \cdot z \cdot \sin(A+B) = \frac{1}{2}zh\sin A + \frac{1}{2}xh\sin B$

$\frac{1}{2}x \cdot z\sin(A+B) = \frac{1}{2}z \cdot x\cos B\sin A + \frac{1}{2}xz\cos A\sin B$

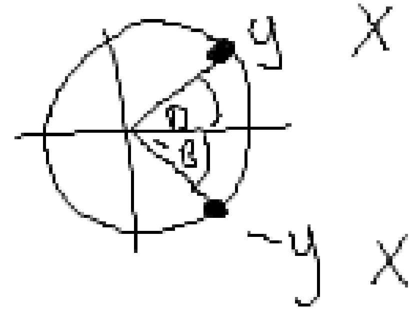
$\frac{1}{2}x \cdot z\left(\underline{\sin(A+B) = \cos B\sin A + \cos A\sin B}\right)$

$$\boxed{\sin(A+B) = \sin A \cos B + \sin B \cos A}$$

$\sin(A-B)$ replace B with $-B$

$\sin(-B) = -\sin(B)$

$\cos(-B) = \cos B$

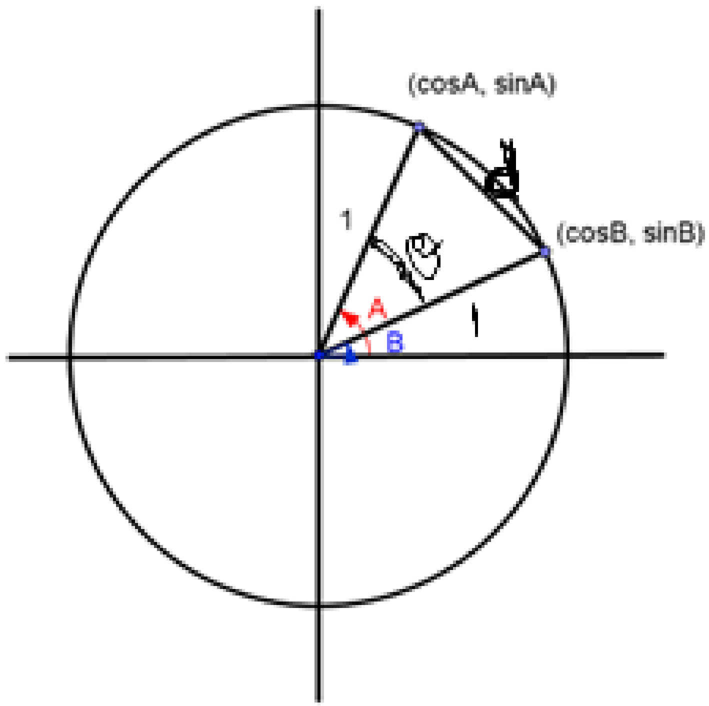


$\downarrow$

$$\sin(A-B) = \sin A \cos(-B) + \sin(-B)\cos A$$

$$\boxed{\sin(A-B) = \sin A \cos B - \sin B \cos A}$$

Cosine:



$\theta = \angle A - \angle B$

Distance formula between two points:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d^2 = (\cos B - \cos A)^2 + (\sin B - \sin A)^2$$

Law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$d^2 = 1^2 + 1^2 - 2(1)(1)\cos(A-B)$$

$$d^2 = 2 - 2\cos(A-B)$$

$$d^2 = d^2$$

$$(\cos B - \cos A)^2 + (\sin B - \sin A)^2 = 2 - 2\cos(A-B)$$

$$\cos^2 B - 2\cos A\cos B + \cos^2 A + \sin^2 B - 2\sin A\sin B + \sin^2 A = 2 - 2\cos(A-B)$$

$$\underline{\cos^2 B} - 2\cos A\cos B + \underline{\cos^2 A} + \underline{\sin^2 B} - 2\sin A\sin B + \underline{\sin^2 A} = 2 - 2\cos(A-B)$$

Pythagorean identity = 1

$$1 + 1 - 2\cos A\cos B - 2\sin A\sin B = 2 - 2\cos(A-B)$$

$$\cancel{2} - 2\cos A\cos B - 2\sin A\sin B = \cancel{2} - 2\cos(A-B)$$

$$-\cancel{2} \qquad\qquad -\cancel{2}$$

$$-2\cos A\cos B - 2\sin A\sin B = -2\cos(A-B)$$

divide by -2

$$\boxed{\cos A\cos B - \sin A\sin B = \cos(A-B)}$$

replace B with $-B$ same as before

$$\boxed{\cos(A+B) = \cos A\cos B - \sin A\sin B}$$

Sum and Difference Identities:

$$\sin(A+B)=\sin A\cos B+\sin B\cos A$$

$$\sin(A-B)=\sin A\cos B-\sin B\cos A$$

$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

Sum and Difference Identities:

$$\sin(A+B)=\sin A\cos B+\sin B\cos A$$

$$\sin(A-B)=\sin A\cos B-\sin B\cos A$$

$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

Examples:

1. Simplify. Write the following expression as a single trigonometric function.

$$\sin 32^\circ \cos 19^\circ - \cos 32^\circ \sin 19^\circ$$

let $A = 32^\circ$

let $B = 19^\circ$

$\sin A \cos B - \underline{\cos A \sin B}$

$= \sin(A-B)$

$= \sin(32-19)$

$= \sin 13^\circ$

2. Calculate the exact value for:

(a) $\cos 15°$

$15° = 45° - 30°$

$\cos(45° - 30°) = \cos 45° \cos 30° + \sin 45° \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$$\boxed{\cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4}}$$

Calculate the exact value for:

(b) $\csc\left(\frac{5\pi}{12}\right) = \csc\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right)$

for the weekend: worry about your test first

HW: pg 306 #1abd,2abd,8

Test Review

* Unit circle values cannot be more than 1 or less than -1
  sinx is never greater than 1 or less than -1

* period of sin|cos is 2π or $360°$
  period of tan is π or $180°$

$$\sin^{-1}(\underbrace{\cos(\pi/3)}_{1/2})$$

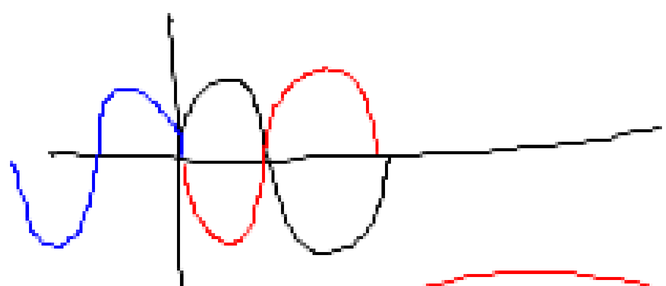
$\sin^{-1}(1/2) = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

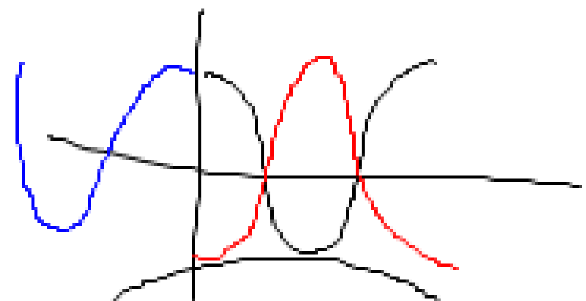
$\sin^{-1}(1/2) =$ where on unit circle is $\sin\theta = 1/2$

$$\cos^{-1}\left(\underbrace{\cos\left(\pi/4\right)}_{\frac{\sqrt{2}}{2}}\right)$$

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} \text{ and } \frac{7\pi}{4}$$

Rx vs Ry





Rx
Ry

Ry = Rx for sin x

Ry ≠ Rx for cos x

terminal arm is ALWAYS POSITIVE (in all quadrants)

if asked for the equation (ex. reflected cosine equation
or
regular sine equation)

use first possible
$HT \geq 0$

AT 80
$(x-80)$ in equation

graph: each block on the graph

$$\frac{\text{label}}{\text{\# of blocks from 0 to that label}}$$

each block: $\frac{\pi}{6} \div 3 = \frac{\pi}{18}$

careful with labels on graphs (doesn't always count up by 1)

$$* \quad 3\sin(\underbrace{2x-80}_{\text{Factored}})+3=0$$

$$3\sin(2(x-40))+3=0$$