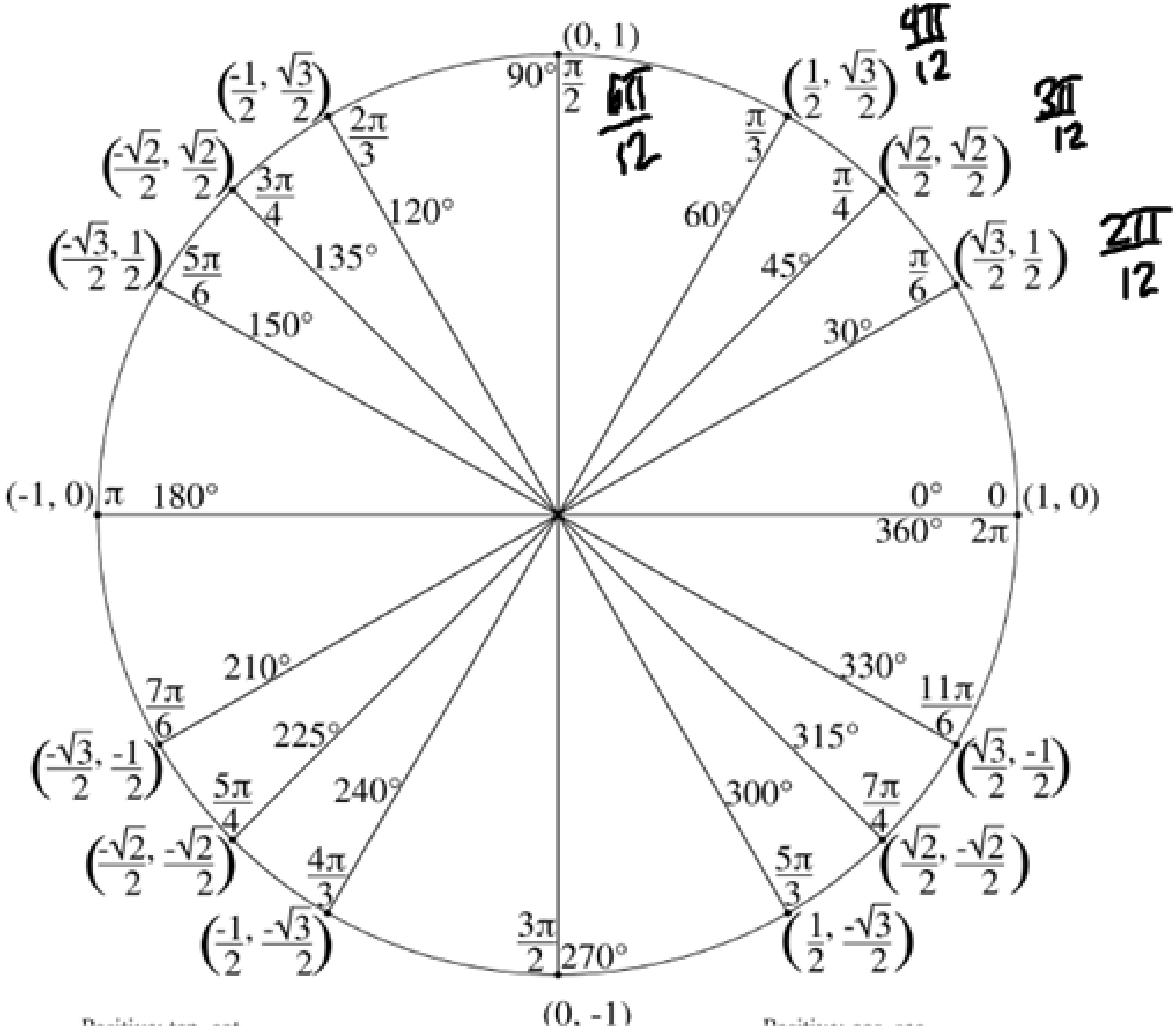


(0, 1)
90°
$\frac{\pi}{2}$
$\frac{6\pi}{12}$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{3}$
60°
$\frac{4\pi}{12}$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\frac{\pi}{4}$
45°
$\frac{3\pi}{12}$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{\pi}{6}$
30°
$\frac{2\pi}{12}$
(1, 0)
0°
0
360°
2π
$\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{2\pi}{3}$
120°
$\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\frac{3\pi}{4}$
135°
$\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{5\pi}{6}$
150°
(-1, 0)
π
180°
210°
$\frac{7\pi}{6}$
$\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$
225°
$\frac{5\pi}{4}$
$\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$
240°
$\frac{4\pi}{3}$
$\left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$
$\frac{3\pi}{2}$
270°
(0, -1)
300°
$\frac{5\pi}{3}$
$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$
315°
$\frac{7\pi}{4}$
$\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$
330°
$\frac{11\pi}{6}$
$\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$

Example:

Find the exact value of $\tan\left(\frac{5\pi}{12}\right)$

① $5\pi/12$

$\downarrow$

$\frac{3\pi}{12} + \frac{2\pi}{12}$

$\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

② $\tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

③ $= \dfrac{\tan\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{6}\right)}{1 - \tan\left(\frac{\pi}{4}\right)\tan\left(\pi/6\right)}$ } this is the identity for tan(A+B)

$$= \frac{1 + \frac{\sqrt{3}}{3}}{1 - 1\left(\frac{\sqrt{3}}{3}\right)} = \frac{\frac{3}{3} + \frac{\sqrt{3}}{3}}{\frac{3}{3} - \frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3-\sqrt{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}}$$

$\longrightarrow$

$$\frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}}$$ *multiply by the conjugate

$$= \frac{9+6\sqrt{3}+3}{9-3}$$

$$= \frac{12+6\sqrt{3}}{6}$$

$$= \boxed{2+\sqrt{3}}$$

$$\boxed{\tan\left(\frac{5\pi}{12}\right) = 2+\sqrt{3}}$$

Double-Angle Identities

note: 2sin A is very different from sin(2A)

1. Use the identity $\sin(A+B)=\sin A\cos B+\cos A\sin B$ to create an identity for $\sin(2A)$.

$$2A = A + A$$

$$\sin(2A) = \sin(A+A)$$

$$= \sin A\cos A + \cos A\sin A$$

$$= 2\sin A\cos A$$

$$\boxed{\sin(2A) = 2\sin(A)\cos(A)}$$

2. Use the identity $\cos(A+B)=\cos A\cos B-\sin A\sin B$ to create an identity for $\cos(2A)$.

$\cos(2A)=\cos(A+A)$

$\cos(2A)=\cos A\cos A-\sin A\sin A$

$\boxed{\cos(2A)=\cos^2 A-\sin^2 A}$

3. Use the Pythagorean identity $\sin^2 A+\cos^2 A=1$ to write an identity for $\cos(2A)$ that contains only the cosine ratio.

$\cos(2A)=\cos^2 A-\underline{\sin^2 A}$

$\sin^2 A+\cos^2 A=1$

$\sin^2 A=1-\cos^2 A$

$\cos(2A)=\cos^2 A-(1-\cos^2 A)$

$\cos(2A)=\cos^2 A-1+\cos^2 A$

$\boxed{\cos(2A)=2\cos^2 A-1}$

4. Write an identity for $\cos(2A)$ that contains only the sine ratio.

$\sin^2 A + \cos^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$

$\cos(2A) = \underline{\cos^2 A} - \sin^2 A$

$\cos(2A) = (1 - \sin^2 A) - \sin^2 A$

$\cos(2A) = 1 - \sin^2 A - \sin^2 A$

$$\boxed{\cos(2A) = 1 - 2\sin^2 A}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$$

Examples:

Given that $\sin\theta = -\frac{5}{13}$ and $\cos\theta = \frac{12}{13}$, find $\tan(2\theta)$.

(1)

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

(2)

$$* \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{-5}{13} \div \frac{12}{13}$$

$$\tan\theta = \frac{-5}{13} \cdot \frac{13}{12}$$

$$\tan\theta = \frac{-5}{12}$$

(3)

$$= \frac{2\left(-5/12\right)}{1-\left(\frac{-5}{12}\right)^2}$$

$$= \frac{-5}{6} \div \left(1 - \frac{25}{144}\right)$$

$$= \frac{-5}{6} \div \left(\frac{144}{144} - \frac{25}{144}\right) = \frac{-5}{6} \div \frac{119}{144} = \frac{-5}{6} \cdot \frac{144}{119} \rightarrow$$

$$\frac{-5}{6}\cdot\frac{144}{119} = \frac{-5}{\cancel{6}}\cdot\frac{\overset{2}{\cancel{12}}\cdot 12}{119} = \frac{-5\cdot 2\cdot 12}{119} = \boxed{\frac{-120}{119}}$$

Pg. 303

Your Turn

Consider the expression $\frac{\sin 2x}{\cos 2x + 1}$.

a) What are the permissible values for the expression?

b) Simplify the expression to one of the three primary trigonometric functions.

c) Verify your answer from part b), in the interval $[0, 2\pi)$, using technology.

$\sin x$
$\cos x$
$\tan x$

A) everything except non permissable values

$\cos(2x) + 1 \neq 0$

$\cos(2x) \neq -1$

$2x \neq \cos^{-1}(-1)$

$2x \neq \pi \pm 2\pi k, k \in I$

$x \neq \frac{\pi}{2} \pm \pi k, k \in I$

$\{x \mid x \neq \frac{\pi}{2} \pm \pi k, k \in I, x \in R\}$

$$\frac{\sin(2x)}{\cos(2x)+1} = \frac{2\sin x\cos x}{\cos(2x)+1} = \frac{2\sin x\cos x}{2\cos^2 x - 1 + 1}$$

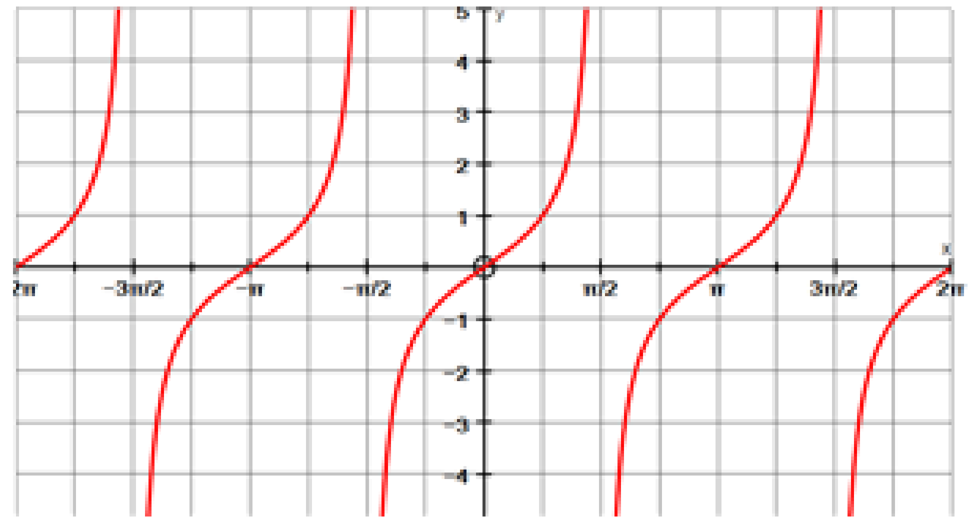
$$= \frac{\cancel{2}\sin x\,\cancel{\cos x}}{\cancel{2}\cos^{\cancel{2}} x}$$

$$= \frac{\sin x}{\cos x}$$

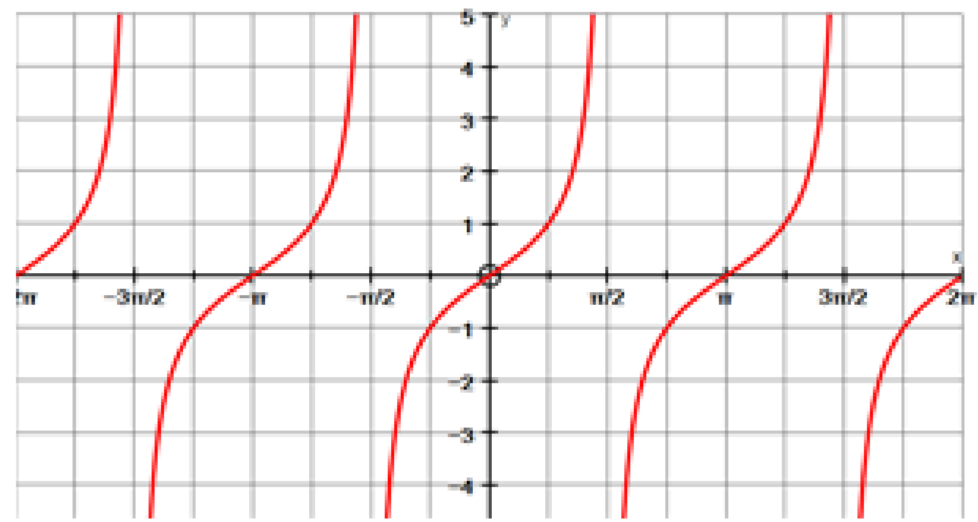
$$= \tan x$$

$$\frac{\sin(2x)}{\cos(2x)+1} = \tan x$$

$y = \tan x$



$y = \dfrac{\sin(2x)}{\cos(2x)+1}$



6.3

Proving Identities

Focus on...

- proving trigonometric identities algebraically
- understanding the difference between verifying and proving an identity
- showing that verifying that the two sides of a potential identity are equal for a given value is insufficient to prove the identity

Examples: Prove the following identities: * we don't know its true yet so no equals sign allowed

1. $\sin x \cos x \cot x = \cos^2 x$

$$\frac{\cancel{\sin x}\cos x \cos x}{\cancel{\sin x}}$$

$\cos^2 x$ ✓ | $\cos^2 x$ ✓

$\sin x \cos x \cot x = \cos^2 x$

*only work on one side

*work on the harder side

2. $\cos x(\sec x - \cos x) = \sin^2 x$

expand

expending

$\cos x \sec x - \cos^2 x$

$\cancel{\cos x} \cdot \frac{1}{\cancel{\cos x}} - \cos^2 x$

$1 - \cos^2 x$

$\sin^2 x$

$\sin^2 x$

$\cos x(\sec x - \cos x) = \sin^2 x$ ✓

3. $\cot^2 x \csc^2 x - \cot^2 x = \cot^4 x$

*common factor

$\cot^2 x(\csc^2 x - 1)$ | $\downarrow$

$\cot^2 x(\cot^2 x)$

$\cot^4 x$ | $\cot^4 x$

$\cot^2 x \csc^2 x - \cot^2 x = \cot^4 x$ ✓

4. $\tan x + \cot x = \sec x \csc x$

$\underbrace{}_{\text{one term}}$

left side has 2 terms so its the harder side to work with first

Goal: one term

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x}$$

$$\frac{1}{\sin x} \cdot \frac{1}{\cos x}$$

$$\csc x \sec x$$

$$\sec x \csc x = \sec x \csc x \checkmark$$

HW: pg 306 #1, 2, 4, 5, 7, 11, 14, 15, 16, 18, 20
and pg 314 #1-4