6.1

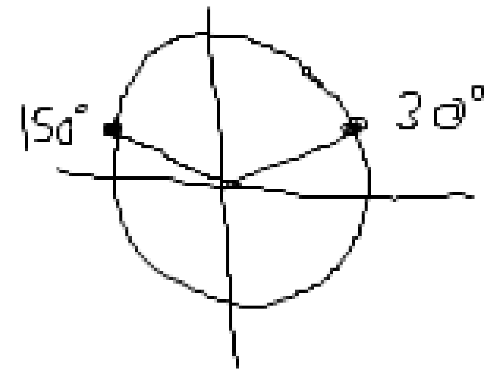
Reciprocal, Quotient, and Pythagorean Identities

Focus on...

- verifying a trigonometric identity numerically and graphically using technology
- exploring reciprocal, quotient, and Pythagorean identities
- determining non-permissible values of trigonometric identities
- explaining the difference between a trigonometric identity and a trigonometric equation

A trigonometric equation is an equation that is true only for certain values of x.

$$\sin x = \frac{1}{2}$$



$$x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$x = \left\{ \begin{matrix} 30° \\ 150° \end{matrix} \pm 360° k,\ k \in I \right\}$$

A trigonometric identity is an equation that is true for all permissible values of x on both sides of the equation.

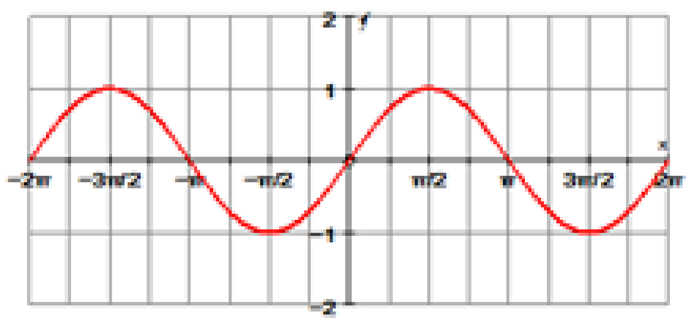
For example:

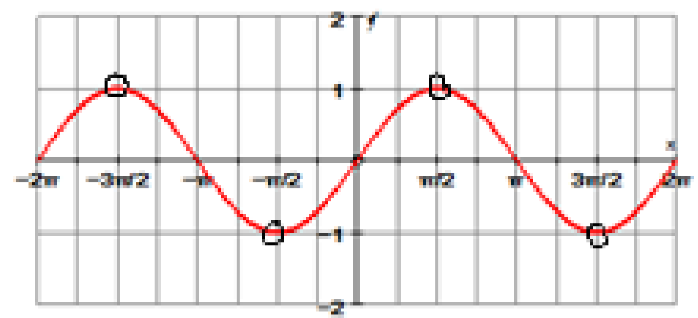
$$\sin x = \cos x \tan x \longrightarrow \sin x = \frac{\cancel{\cos x}}{1}\,\frac{\sin x}{\cancel{\cos x}} \rightarrow \sin x = \sin x$$

(a) Graph the curves $y = \sin x$ and $y = \cos x \tan x$. What do you notice?

non permissable values
when $\cos x = 0$

sin =





(b) What are the non-permissible values of x in the equation $\sin x = \cos x \tan x$?

can't divide by 0 so whenever you see dividing set the denominator = 0 and thats a non permissable value.

$\sin x = \cos x \left(\frac{\sin x}{\cos x}\right)$

$\cos x \neq 0$

$x \neq \cos^{-1}(0)$

$\boxed{x \neq \frac{\pi}{2}, \frac{3\pi}{2} \pm 2\pi k, k \in I}$

(c) Verify that $60°$ and $x = \frac{\pi}{4}$ are solutions of the equation.

$\sin 60° = \cos 60° \tan 60°$

$\frac{\sqrt{3}}{2} = \frac{1}{2}(\sqrt{3})$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ ✓

$\sin(\pi/4) = \cos(\pi/4)\tan(\pi/4)$

$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}(1)$

$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$ ✓

Reciprocal Identities:

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Quotient Identities:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

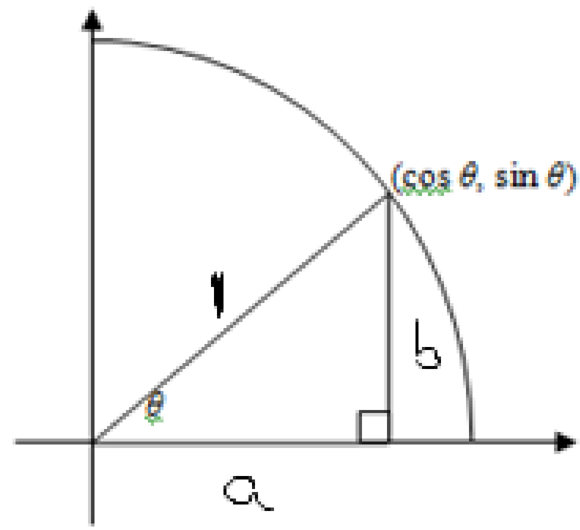
Pythagorean Identities:

$$(\cos\theta)^2 = \cos^2\theta$$

$$a^2 + b^2 = c^2$$

$$\cos^2\theta + \sin^2\theta = 1^2$$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$



$$\cos^2(\pi/6) + \sin^2(\pi/6) = 1$$

$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$$

$$\frac{3}{4} + \frac{1}{4} = 1$$

$$1 = 1 \checkmark$$

Divide every term in the identity $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$.
Simplify.

$$\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

Divide every term in the identity $\sin^2\theta + \cos^2\theta = 1$ by $\sin^2\theta$.
Simplify.

$$\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

The three forms of the Pythagorean identity are:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

Extend

15. Given $\csc^2 x + \sin^2 x = 7.89$, find the value of $\frac{1}{\csc^2 x} + \frac{1}{\sin^2 x}$.

$$\csc^2 x = \frac{1}{\sin^2 x}$$

$$\sin^2 x = \frac{1}{\csc^2 x}$$

easy

$$\sin^2 x + \csc^2 x = 7.89$$

hard

common denominator

$$\frac{\sin^2 x}{\csc^2 x \cancel{\sin^2 x}} + \frac{\csc^2 x}{\csc^2 x \sin^2 x} = \frac{\sin^2 x + \csc^2 x}{\csc^2 x \sin^2 x} = \frac{7.89}{\csc^2 x \sin^2 x}$$

$$\frac{7.89}{\csc^2 x \sin^2 x} = \frac{7.89}{\frac{1}{\sin^2 x} \cdot \sin^2 x} = \frac{7.89}{\left(\frac{\sin^2 x}{\sin^2 x}\right)} = \frac{7.89}{1} = 7.89 \checkmark$$

HW: pg 296 #1,3,4,5,6,11,14,17,