

Additional Trig Equations

① A) $\cos x (2 \cos x + \sqrt{2}) = 0$

$\cos x = 0$

~~$\cos x = 0$~~ $2 \cos x + \sqrt{2} = 0$

$\cos x = -\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{2} + 2\pi n$

$x = \frac{3\pi}{2} + 2\pi n$

$x = \frac{3\pi}{4} + 2\pi n, \quad x = \frac{5\pi}{4} + 2\pi n$

For $x \in [-\pi, 2\pi]$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{3\pi}{4}, -\frac{\pi}{4}, -\frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{5\pi}{4}$

B) $\cos^2 x \sin 3x - \frac{1}{2} \sin 3x = 0$

$\sin 3x (\cos^2 x - \frac{1}{2}) = 0$

$\sin 3x = 0$

$\cos^2 x = \frac{1}{2}$

$3x = 0 + \pi n$

$x = 0 + \frac{\pi n}{3}, n \in \mathbb{I}$

$\cos x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4} + \pi n$

$x = \frac{7\pi}{4} + \pi n$

$x = \frac{3\pi}{4} + \pi n$

$x = \frac{5\pi}{4} + \pi n$

$n \in \mathbb{I}$

$$\begin{aligned}
 \text{C) } \cos^2 x - \cos x &= 0 \\
 \cos x (\cos x - 1) &= 0 \\
 \cos x = 0 & \qquad \cos x - 1 = 0 \\
 \downarrow & \qquad \qquad \downarrow \\
 x = 90^\circ + 180^\circ n & \qquad x = 0^\circ + 360^\circ n \qquad n \in \mathbb{Z}
 \end{aligned}$$

$$x \in [-720^\circ, 0^\circ]$$

$$\begin{aligned}
 x = -90^\circ, -270^\circ, -450^\circ, -630^\circ, \\
 0^\circ, -360^\circ, -720^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{D) } 3 \sin^3 x - 2 \sin^2 x - 6 \sin x + 4 &= 0 \\
 \sin^2 x (3 \sin x - 2) - 2(3 \sin x - 2) &= 0 \\
 (3 \sin x - 2)(\sin^2 x - 2) &= 0
 \end{aligned}$$

$$3 \sin x - 2 = 0$$

$$\sin x = \frac{2}{3}$$

$$x = 0.7297$$

$$x = \pi - 0.7297 = 2.4119$$

$$\sin^2 x - 2 = 0$$

$$\sin^2 x = 2$$

$$\sin x = \pm \sqrt{2}$$

no solution

$$\text{all solutions: } x = 0.7297 + 2\pi n$$

$$x = 2.4119 + 2\pi n$$

$$x \in [0, 3\pi)$$

$$x = 0.73, 7.01, 2.41, 8.70$$

$$\textcircled{1} \text{ E) } 2 \sin^2 x - 7x + 3 = 0$$

$$(2 \sin x - 1)(\sin x - 3) = 0$$

$$\sin x = \frac{1}{2}$$

$$\sin x = 3 \rightarrow \text{No solution}$$

$$\boxed{\begin{array}{l} x = 30^\circ + 360^\circ n \\ x = 150^\circ + 360^\circ n \end{array}} \quad \left. \vphantom{\begin{array}{l} x = 30^\circ + 360^\circ n \\ x = 150^\circ + 360^\circ n \end{array}} \right\} n \in \mathbb{I}$$

$$\text{F) } \sin^4 x = \frac{1}{16}$$

$$\sin^2 x = \pm \frac{1}{4}$$

$$\sin x = \pm \frac{1}{2}$$

$$x = 30^\circ + 360^\circ n$$

$$x = 150^\circ + 360^\circ n$$

$$x = 210^\circ + 360^\circ n$$

$$x = 330^\circ + 360^\circ n$$

$$\left. \vphantom{\begin{array}{l} x = 30^\circ + 360^\circ n \\ x = 150^\circ + 360^\circ n \\ x = 210^\circ + 360^\circ n \\ x = 330^\circ + 360^\circ n \end{array}} \right\} n \in \mathbb{I}$$

$$\text{G) } 3 \sin \frac{1}{3} x \cos x - \sin \frac{1}{3} x = 0$$

$$\sin \frac{1}{3} x (3 \cos x - 1) = 0$$

$$\sin \frac{1}{3} x = 0$$

$$\frac{1}{3} x = 0^\circ + 360^\circ n$$

$$\frac{1}{3} x = 180^\circ + 360^\circ n$$

$$\boxed{\begin{array}{l} x = 0^\circ + 1080^\circ n \\ x = 540^\circ + 1080^\circ n \end{array}} \quad n \in \mathbb{I}$$

$$3 \cos x - 1 = 0$$

$$\cos x = \frac{1}{3}$$

$$\boxed{x = 70.53^\circ + 360^\circ n}$$

$$x = 360^\circ - 70.53^\circ$$

$$\boxed{x = 289.47^\circ + 360^\circ n}$$

$$\textcircled{2} \text{ A) } 2 \sin(x+60^\circ) = 1$$

$$\sin(x+60^\circ) = \frac{1}{2}$$

2 solutions for $[0, 360^\circ]$
6 solutions for $[-540^\circ, 540^\circ]$

$$x+60^\circ = 30^\circ + 360^\circ n$$

$$x = -30^\circ + 360^\circ n$$

$$x+60^\circ = 150^\circ + 360^\circ n$$

$$x = 90^\circ + 360^\circ n$$

For $x \in [-540^\circ, 540^\circ]$

$$x = -30^\circ, -390^\circ, 330^\circ,$$

$$90^\circ, -270^\circ, 450^\circ$$

$$\textcircled{3} \text{ B) } 2 \sin(2(x-60^\circ)) = -\sqrt{3}$$

$$\sin(2(x-60^\circ)) = -\frac{\sqrt{3}}{2}$$

2 solutions for $[0, 180^\circ]$
5 solutions for $[0, 360^\circ]$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\theta = 240^\circ + 360^\circ n$$

$$\theta = 300^\circ + 360^\circ n$$

$$2(x-60^\circ) = 240^\circ + 360^\circ n$$

$$x-60^\circ = 120^\circ + 180^\circ n$$

$$x = 180^\circ + 180^\circ n$$

$$2(x-60^\circ) = 300^\circ + 360^\circ n$$

$$x-60^\circ = 150^\circ + 180^\circ n$$

$$x = 210^\circ + 180^\circ n$$

For $x \in [0, 360^\circ]$

$$x = 180^\circ, 0^\circ, 360^\circ,$$

$$210^\circ, 30^\circ$$

② c) $\cos(3x - 120^\circ) = \frac{1}{2}$

2 solutions for $[0, 120^\circ]$
6 solutions for $[-360, 0]$

$$3x - 120^\circ = 60^\circ + 360^\circ n$$

$$3x = 180^\circ + 360^\circ n$$

$$x = 60^\circ + 120^\circ n$$

$$3x - 120^\circ = 300^\circ + 360^\circ n$$

$$3x = 420^\circ + 360^\circ n$$

$$x = 140^\circ + 120^\circ n$$

for $x \in [-360^\circ, 0^\circ]$

$x = -60^\circ, -180^\circ, \overset{-300^\circ}{-240^\circ}, -100^\circ, -220^\circ, -340^\circ$

D) $2 \sin\left[3\left(x - \frac{\pi}{2}\right)\right] = 1$

2 solutions for $[0, \frac{\pi}{3}]$
9 solutions for $[-\pi, 2\pi]$

$$\sin\left[3\left(x - \frac{\pi}{2}\right)\right] = \frac{1}{2}$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6} + 2\pi n$$

$$\theta = \frac{5\pi}{6} + 2\pi n$$

$$3\left(x - \frac{\pi}{2}\right) = \frac{\pi}{6} + 2\pi n$$

$$3\left(x - \frac{\pi}{2}\right) = \frac{5\pi}{6} + 2\pi n$$

$$x - \frac{\pi}{2} = \frac{\pi}{18} + \frac{2\pi n}{3}$$

$$x - \frac{\pi}{2} = \frac{5\pi}{18} + \frac{2\pi n}{3}$$

$$x = \frac{\pi}{18} + \frac{9\pi}{18} + \frac{2\pi n}{3}$$

$$x = \frac{5\pi}{18} + \frac{9\pi}{18} + \frac{2\pi n}{3}$$

$$x = \frac{10\pi}{18} + \frac{2\pi n}{3}$$

$$x = \frac{14\pi}{18} + \frac{2\pi n}{3}$$

$$x = \frac{5\pi}{9} + \frac{6\pi n}{9}$$

$$x = \frac{7\pi}{9} + \frac{6\pi n}{9}$$

For $x \in [-\pi, 2\pi]$

$x = \frac{5\pi}{9}, \frac{11\pi}{9}, \frac{17\pi}{9}, \frac{23\pi}{9}, -\frac{7\pi}{9}, 0$

$\frac{7\pi}{9}, \frac{13\pi}{9}, \frac{\pi}{9}, -\frac{5\pi}{9}$

$$(2) F) 4 \cos \left[\frac{1}{2} \left(x - \frac{\pi}{3} \right) \right] = 8$$

$$\cos \left[\frac{1}{2} \left(x - \frac{\pi}{3} \right) \right] = 2$$

NO solution

$$G) \cos \left(\frac{1}{2} (x - 5^\circ) \right) = -\frac{1}{2}$$

2 solutions for $[0, 720^\circ]$

0 solutions $[-180^\circ, 180^\circ]$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = 120^\circ + 360n$$

$$\frac{1}{2}(x - 5^\circ) = 120^\circ + 360n$$

$$x - 5^\circ = 240^\circ + 720n$$

$$x = 245^\circ + 720n$$

$$\theta = 240^\circ + 360n$$

$$\frac{1}{2}(x - 5^\circ) = 240^\circ + 360n$$

$$x - 5^\circ = 480^\circ + 720n$$

$$x = 485^\circ + 720n$$

NO solutions in $x \in [-180^\circ, 180^\circ]$

$$H) \frac{3}{2} = 2 + 3 \cos 4 \left(x + \frac{\pi}{6} \right)$$

2 solutions for $[0, \frac{\pi}{2}]$

$$-\frac{1}{2} = 3 \cos 4 \left(x + \frac{\pi}{6} \right)$$

16 solutions $[-\pi, 3\pi]$

$$-\frac{1}{6} = \cos \left(4 \left(x + \frac{\pi}{6} \right) \right)$$

$$4 \left(x + \frac{\pi}{6} \right) = \arccos(-1/6) = 1.7382 + 2\pi n$$

$$4 \left(x + \frac{\pi}{6} \right) = 2\pi - 1.7382$$

$$x + \frac{\pi}{6} = 0.43455 + \frac{\pi}{2} n$$

$$4 \left(x + \frac{\pi}{6} \right) = 4.5449 + 2\pi n$$

$$x = -0.089 + \frac{\pi}{2} n$$

$$x + \frac{\pi}{6} = 1.1362 + \frac{\pi}{2} n$$

$$x = -0.089, -1.66, 1.48, 3.05,$$

$$x = 0.61 + \frac{\pi}{2} n$$

$$4.62, 6.19, 7.76, 9.33,$$

$$0.61, -0.96, -2.53, 2.18, 3.75, 5.32, 6.89, 8.46$$

$$\textcircled{B} \text{ A) } 0 = 2 \sin x - 1$$

$$\frac{1}{2} = \sin x$$

$$x = 30^\circ + 360n$$

$$x = 150^\circ + 360n$$

$$n \in \mathbb{I}$$

$$\text{B) } 0 = \cos(3x) - 1$$

$$1 = \cos(3x)$$

$$3x = 0^\circ + 360n$$

$$x = 0^\circ + 120^\circ n$$

$$n \in \mathbb{I}$$

$$\text{C) } 0 = 2 \sin x + 4$$

$$-2 = \sin x$$

NO solution (no x -val)