Pg. 261

Your Turn

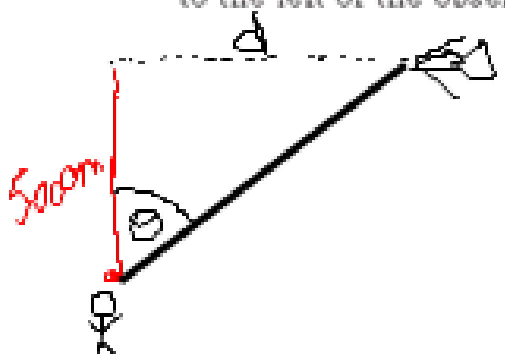
A small plane is flying at a constant altitude of 5000 m directly toward an observer. Assume the ground is flat in the region close to the observer.

height

a) Sketch the graph of the function that represents the relation between the horizontal distance, in metres, from the observer to the plane and the angle, in degrees, formed by the vertical and the line of sight to the plane.

b) Use the characteristics of the tangent function to describe what happens to the graph as the plane flies from the right of the observer to the left of the observer.

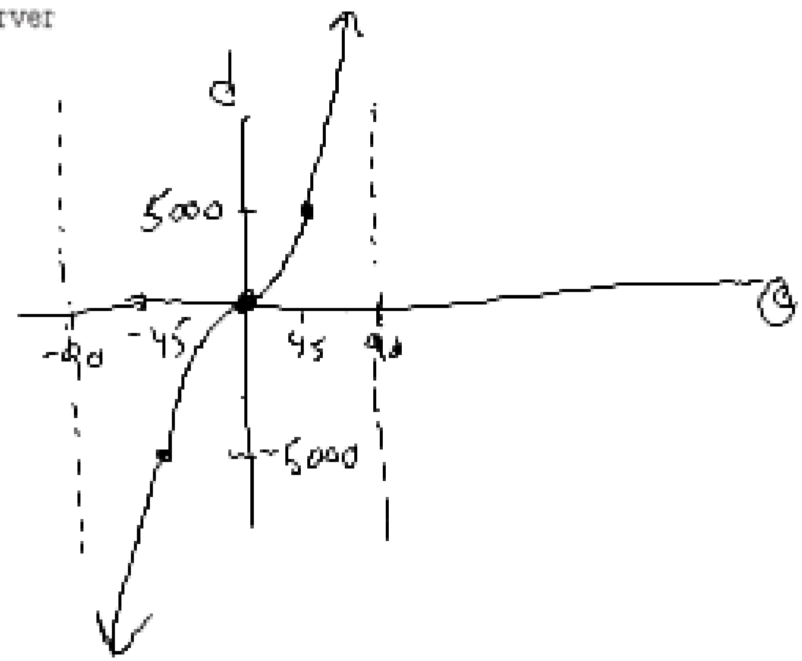
graph goes to negative d values to represent that the plane is now to our left.



A
O } tan

$$\tan\theta = \frac{d}{5000}$$

$$5000\tan\theta = d$$



5000 = VS

period of 180

VA = 90

5.4

Equations and Graphs of Trigonometric Functions

Focus on...

- using the graphs of trigonometric functions to solve equations
- analysing a trigonometric function to solve a problem
- determining a trigonometric function that models a problem
- using a model of a trigonometric function for a real-world situation

Solving Trigonometric Equations

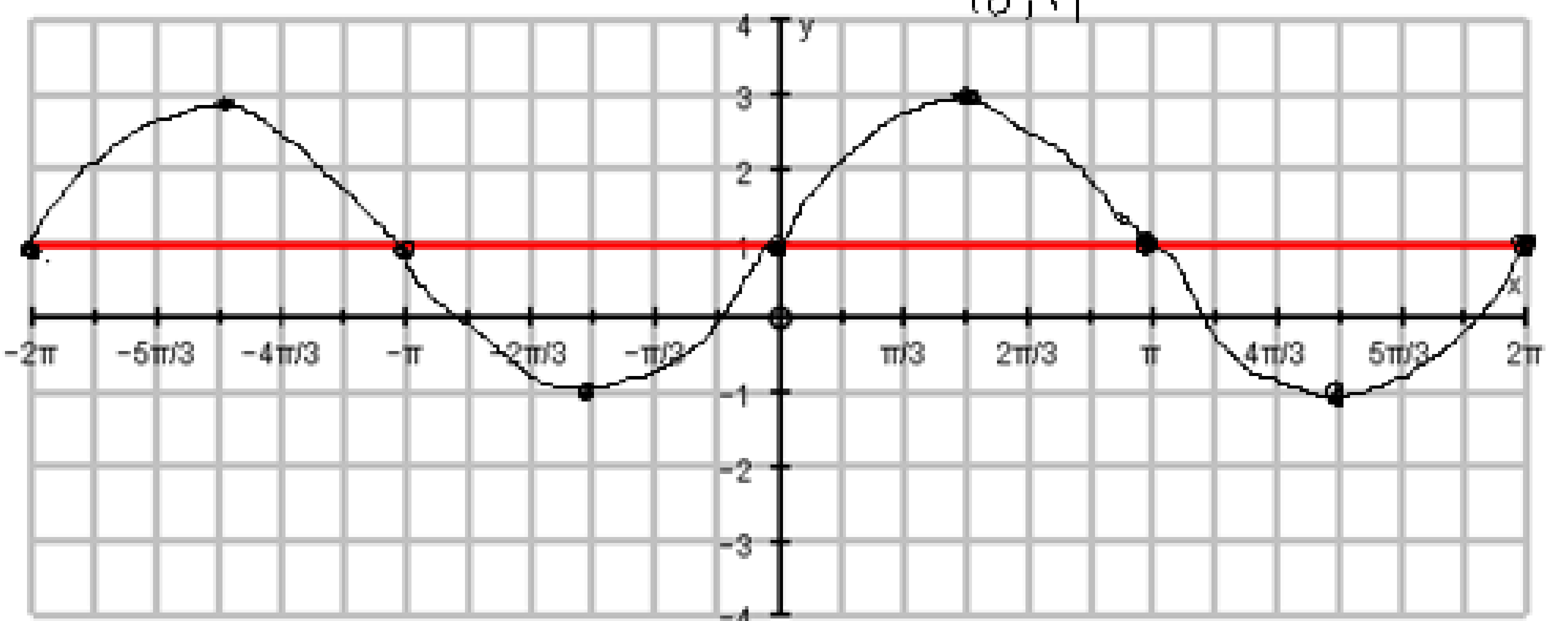
Solve both graphically and algebraically the equation $2\sin x + 1 = 1$ for $-2\pi \le x \le 2\pi$.

Graph the line $y = 2\sin x + 1$. Find the points where $y = 1$.

VS 2 VT 1 (S.A y=1)   sin: S.A max S.A min S.A
(0,1)

Period
2π

Key points
$\frac{2\pi}{4} = \frac{\pi}{2}$



y=1

each block is π/6

The answer is the x values where the two curves meet

$x = \{-2\pi, -\pi, 0, \pi, 2\pi\}$

Can also solve algebraically:

$$2\sin x + 1 = 1 \qquad -2\pi \le x \le 2\pi$$

$2\sin x = 0$

$\sin x = 0$ when does $\sin x = 0$ on unit circle

$x = 0$ $\qquad$ $x = \pi$

use coterminal angles to find all answers b/n -2π and 2π

$0 + 2\pi = 2\pi$ $\qquad$ $\pi + 2\pi = \cancel{3\pi}$ not in domain

$0 - 2\pi = -2\pi$ $\qquad$ $\pi - 2\pi = -\pi$

$$x = \{-2\pi, -\pi, 0, \pi, 2\pi\}$$

Examples:

$2\cos(2(x+15°)) = -2$

must be factored

1. Solve graphically: $2\cos(2x+30°) = -2$, general solution.

Verify by solving algebraically.

$y = 2\cos(2(x+15°))$

$y = -2$

VS 2 HS $\frac{1}{2}$ HT $-15°$ VT 0

(S.A $y=0$)

period

HS $\cdot$ 360

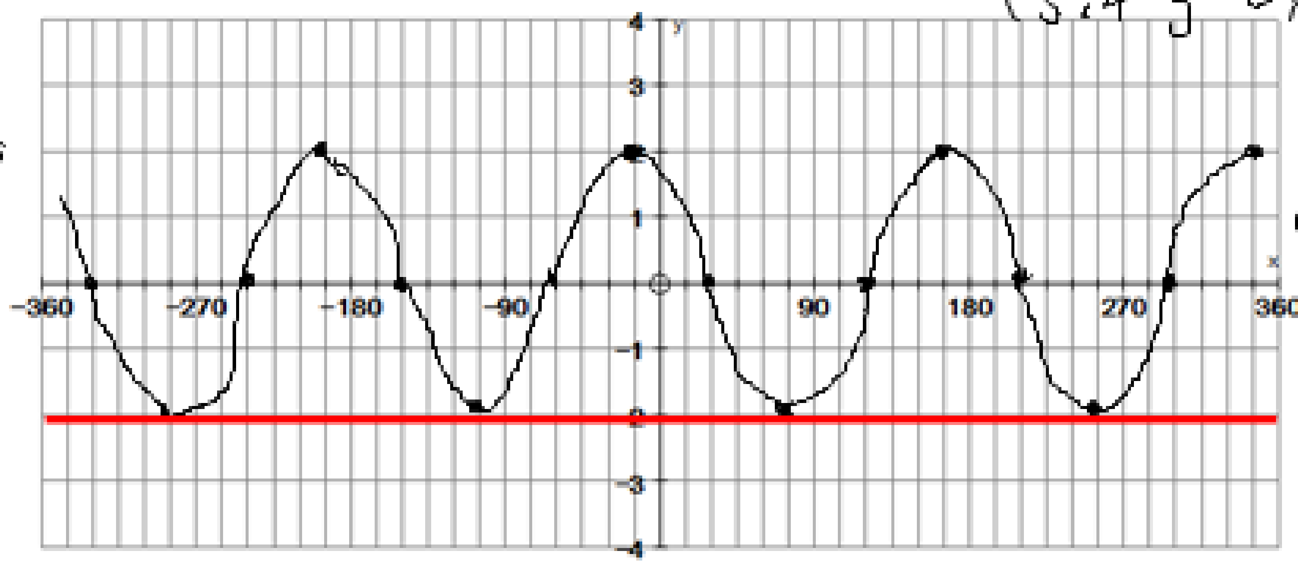
$\frac{1}{2} \cdot 360°$

180°

key pts

$180° \div 4$

45°



cos

max S.A min S.A

$(-15, 2)$ $(30, 0)$

Max

$90° \div 6 = 15°$ (each block is 15°)

All general solutions: just list the ones in the first period then apply general solution.

$\boxed{x = 75° \pm 180°k, k \in I}$

$2\cos(\underbrace{2x+30°}) = -2$

all solutions in degrees

let $a = 2x+30$

$2\cos a = -2$

$\cos a = -1$ on unit circle when does $\cos = -1$

$a = 180° \pm 360°k, k \in I$

$2x+30 = 180° \pm 360°k, k \in I$
$\quad -30 \qquad -30$

$\frac{2x}{2} = \frac{150°}{2} \pm \frac{360°}{2}k, k \in I$

$\boxed{x = 75° \pm 180°k, k \in I}$

only HS affects the period

All values in degrees

B) $\underbrace{\sin(\frac{1}{2}x+20°)}_{a = \frac{1}{2}x + 20°} = \frac{-\sqrt{2}}{2}$

$$\sin a = \frac{-\sqrt{2}}{2}$$

$$a = \begin{array}{l} 225° \\ 315° \end{array} \pm 360°k, k \in I$$

$$\frac{1}{2}x + 20° = \begin{array}{l} 225°\ -20° \\ 315° \end{array} \pm 360°k, k \in I$$

$$-20 \qquad\qquad -20$$

$$2 \times \frac{1}{2}x = \begin{array}{l} 205° \times 2 \\ 295° \times 2 \end{array} \pm 360° \times 2\, k, k \in I \longrightarrow \boxed{x = \begin{array}{l} 410° \\ 590° \end{array} \pm 720°k, k \in I}$$

b) $\cos^2 x = \cos x \quad x \in [-2\pi, 4\pi)$ $\quad x \in \left[-\frac{4\pi}{2}, \frac{8\pi}{2}\right)$

$\cos^2 x - \cos x = 0$

$\cos x (\cos x - 1) = 0$

$\cos x = 0$

$x = \frac{\pi}{2}$

$\frac{3\pi}{2}$

$\cos x - 1 = 0$

$\cos x = 1$

$x = 0$

$x = \left.\begin{matrix} 0 \\ \pi/2 \\ 3\pi/2 \end{matrix}\right. \pm 2\pi k, \; k \in \mathbb{Z}$

$0 + 2\pi = 2\pi$

$2\pi + 2\pi = \cancel{4\pi}$

$0 - 2\pi = -2\pi$

$\frac{\pi}{2} + 2\pi = \frac{5\pi}{2}$

$\frac{\pi}{2} - 2\pi = -\frac{3\pi}{2}$

$\frac{3\pi}{2} + 2\pi = \frac{7\pi}{2}$

$\frac{3\pi}{2} - 2\pi = -\frac{\pi}{2}$

$x = \left\{-2\pi, -\frac{3\pi}{2}, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, 2\pi\right\}$

HW: Worksheet and Pg 275 #5