5.3

The Tangent Function

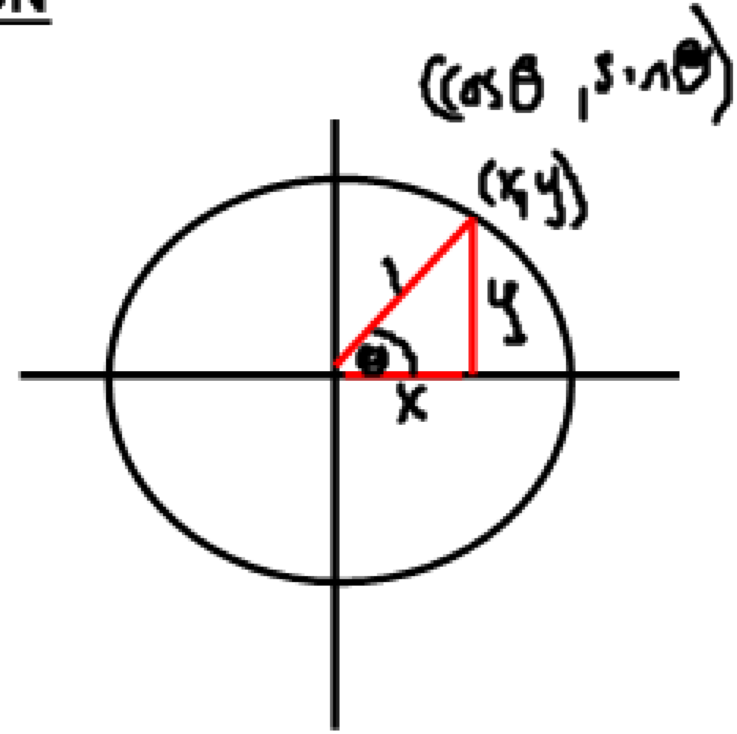
Focus on…

- sketching the graph of $y = \tan x$
- determining the amplitude, domain, range, and period of $y = \tan x$
- determining the asymptotes and x-intercepts for the graph of $y = \tan x$
- solving a problem by analysing the graph of the tangent function

TANGENT FUNCTION

Recall: $\tan\theta = \dfrac{Opp}{Adj}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$



$$\sin\theta = \frac{O}{H} = \frac{y}{1} = y$$

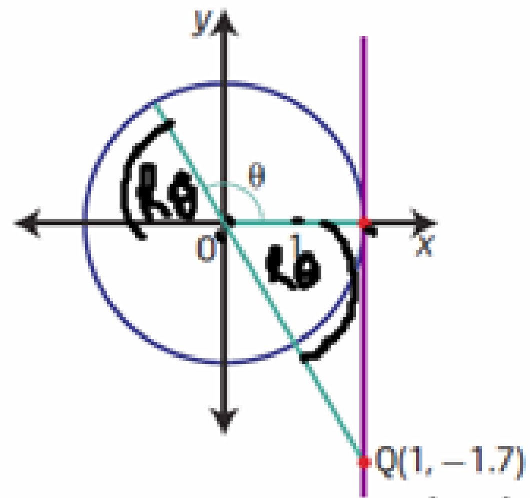
$$\cos\theta = \frac{A}{H} = \frac{x}{1} = x$$

$$\tan\theta = \frac{O}{A} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

Practise

1. For each diagram, determine tan θ and the value of θ, in degrees. Express your answer to the nearest tenth, when necessary.

b)



x, y

tan unit circle values:

1

−1

$\pm\sqrt{3}$

$\pm\frac{\sqrt{3}}{3}$

$\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-1.7}{1}$

$\boxed{\tan\theta = -1.7}$

$\theta = \tan^{-1}(-1.7)$ *degrees

$\theta = -59.5°$ (Ref angle)

Q II using a Ref angle of 59.5°

$180° - 59.5° = \boxed{120.5°}$

All angles should be positive: $-\frac{\pi}{3} + \frac{6\pi}{3} = \frac{5\pi}{3}$

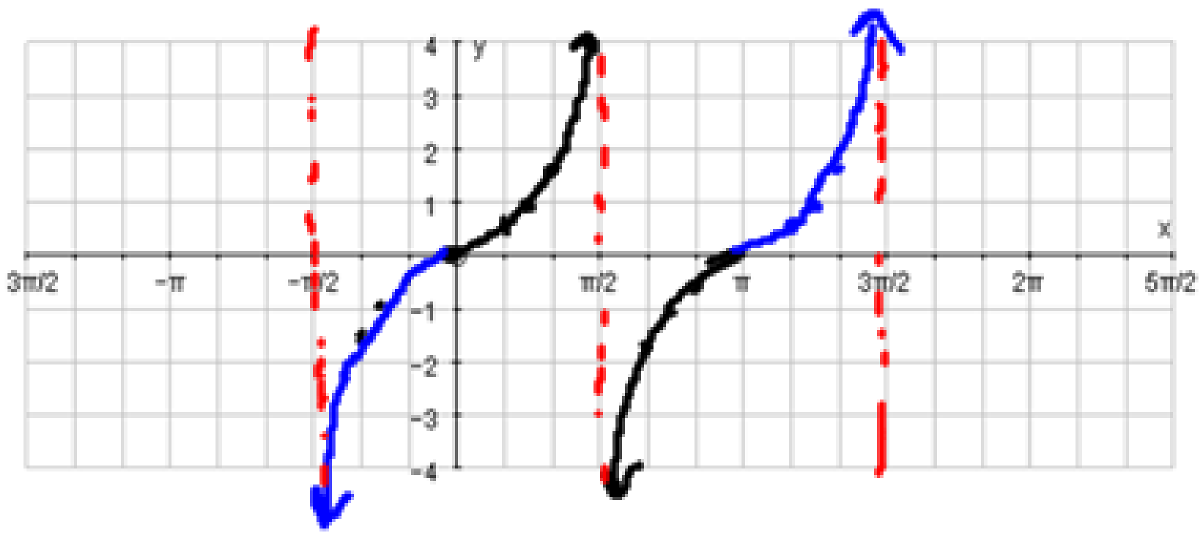
$$\theta = \begin{cases} -\frac{\pi}{3} \\ \frac{7\pi}{6} \end{cases} \pm 2\pi k, k \in I$$

$$\theta = \begin{cases} 5\pi/3 \\ 7\pi/6 \end{cases} \pm 2\pi k, k \in I$$

The Graph of $y = \tan x$

Complete the following table of values:

x (θ)	y	x (θ)	y
0	0	$\frac{7\pi}{6}$	0.6
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3}=0.57$	$\frac{5\pi}{4}$	1
$\frac{\pi}{4}$	1	$\frac{4\pi}{3}$	1.7
$\frac{\pi}{3}$	$\sqrt{3}=1.7$	$\frac{3\pi}{2}$	undef.
$\frac{\pi}{2}$	undef.	$\frac{5\pi}{3}$	-1.7
$\frac{2\pi}{3}$	-1.7	$\frac{7\pi}{4}$	-1
$\frac{3\pi}{4}$	-1	$\frac{11\pi}{6}$	-0.6
$\frac{5\pi}{6}$	-0.6	2π	0
π	0		



period of tan: π

undefined places are called V.A (vertical asymptotes)

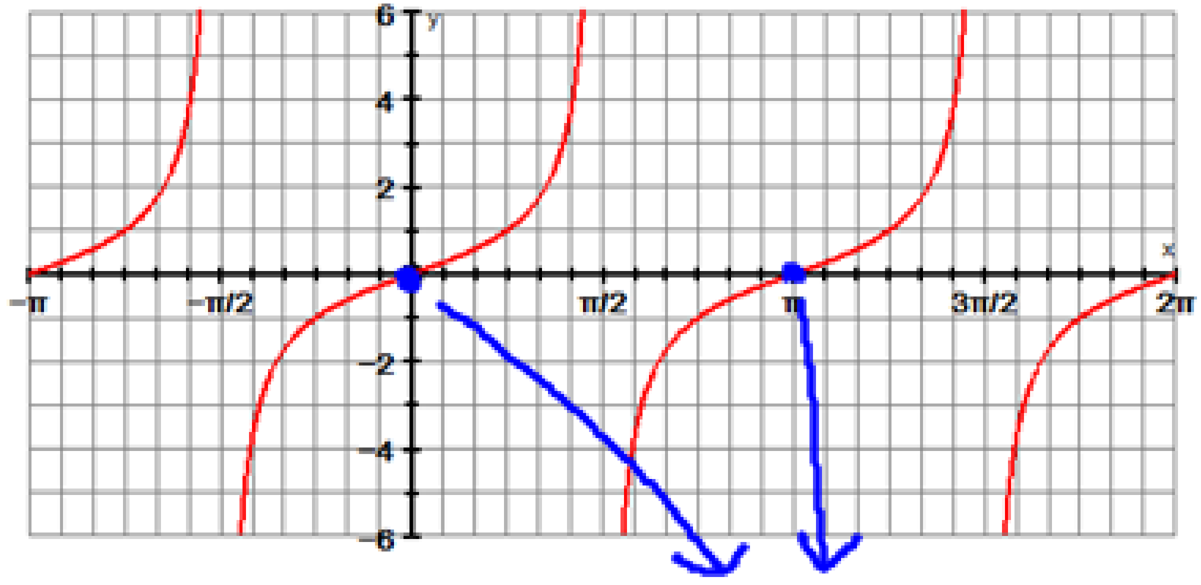
↳ approaches but never actually becomes straight up + down

Amplitude: none

Period: π

Range: $\{y \mid y \in R\}$

Vertical Asymptotes: $\{\frac{\pi}{2} \pm \pi k, k \in I\}$

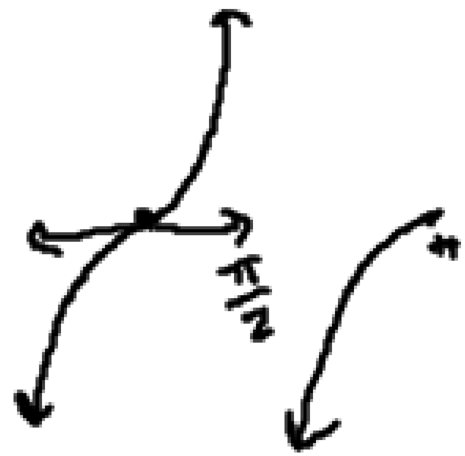


Domain:

$\{x \mid x \neq \frac{\pi}{2} \pm \pi k, k \in I, x \in R\}$

all values except the V.A

Transformations of the tangent function:



$y = a\tan(b(x-c)) + d$

a – vertical stretch

$\frac{1}{b}$ -horizontal stretch

c – horizontal translation

d – vertical translation

when we transform a tangent function

- move the inflection point the way you would move the vertex of a parabola (it is only affected by HT and VT)
- Draw in VA by going $\frac{1}{2}$period to the left and $\frac{1}{2}$ period to the right
- Other key points are halfway between inflection pt and VA, up by the VS

$$y = 2\tan\left(x - \frac{\pi}{2}\right)$$

* Starting pt (HT, VT)

* HT ± $\frac{\text{period}}{2}$ = V. A

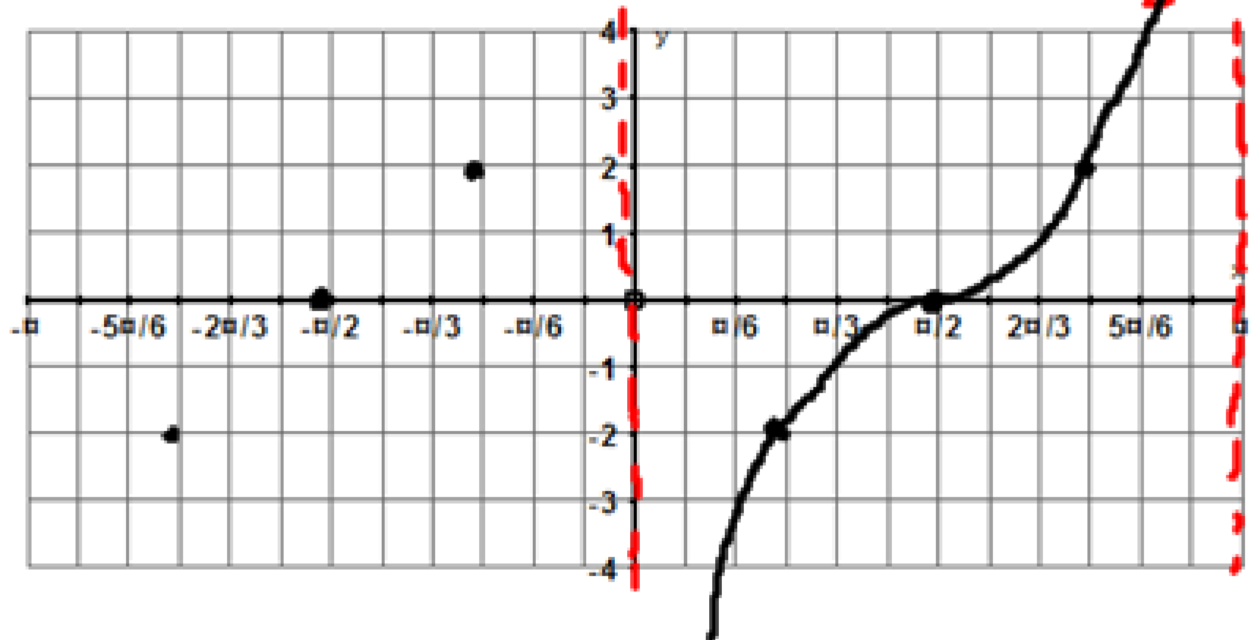
$\frac{\pi}{2} + \frac{\pi}{2} = \pi$

$\frac{\pi}{2} - \frac{\pi}{2} = 0$

List transformations

VS 2

HT $\pi/2$



Period: $HS \cdot \pi$

$= 1 \cdot \pi$

$= \pi$

Domain:

$\{x \mid x \neq 0 \pm \pi k, k \in I, x \in R\}$

Range:

$\{y \mid y \in R\}$

$y = \tan\left(2\left(x + \frac{\pi}{4}\right)\right)$

Starting pt (HT, VT) $\left(-\frac{\pi}{4}, 0\right)$

List transformations

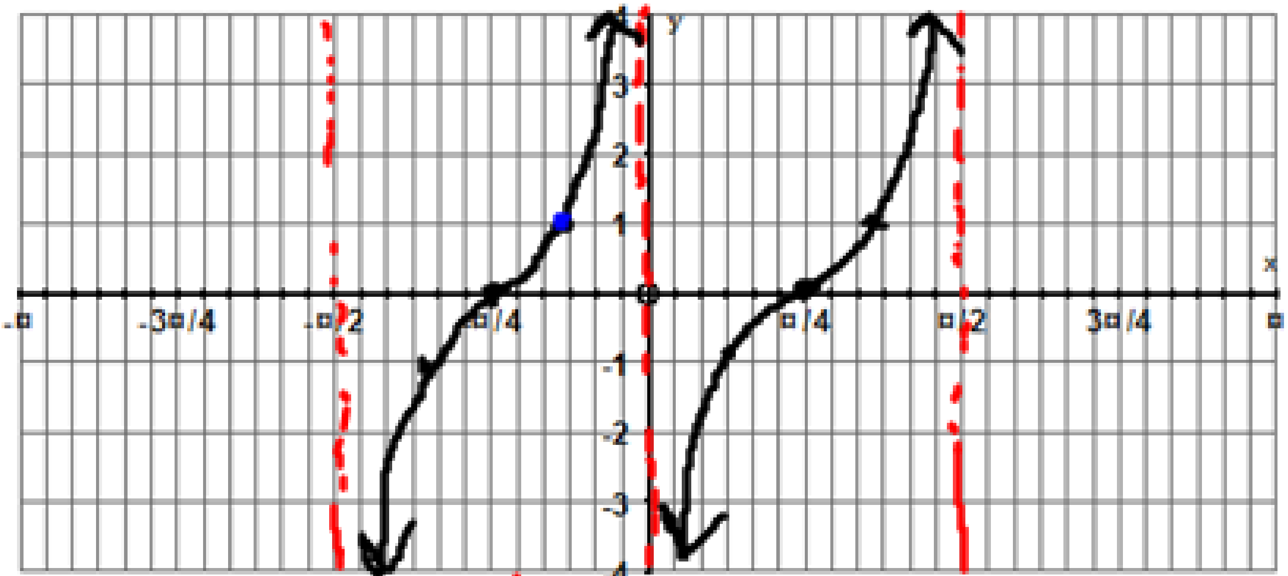
HS = 1/2

HT = $-\frac{\pi}{4}$

Period: = HS · π

= $\frac{1}{2} \cdot \pi$

= π/2



V.A = HT ± $\frac{\text{Period}}{2}$ = $-\frac{\pi}{4} \pm \frac{\pi}{4}$ = $0, -\frac{\pi}{2}$, etc

Domain: $\{x \mid x \neq 0 \pm \frac{\pi}{2}k, k \in \mathbb{Z}, x \in \mathbb{R}\}$

Range: $\{y \mid y \in \mathbb{R}\}$

$$y = \frac{1}{2}\tan\left(\left(x + \frac{\pi}{4}\right)\right) + 2$$

starting pt $\left(-\frac{\pi}{4}, 2\right)$

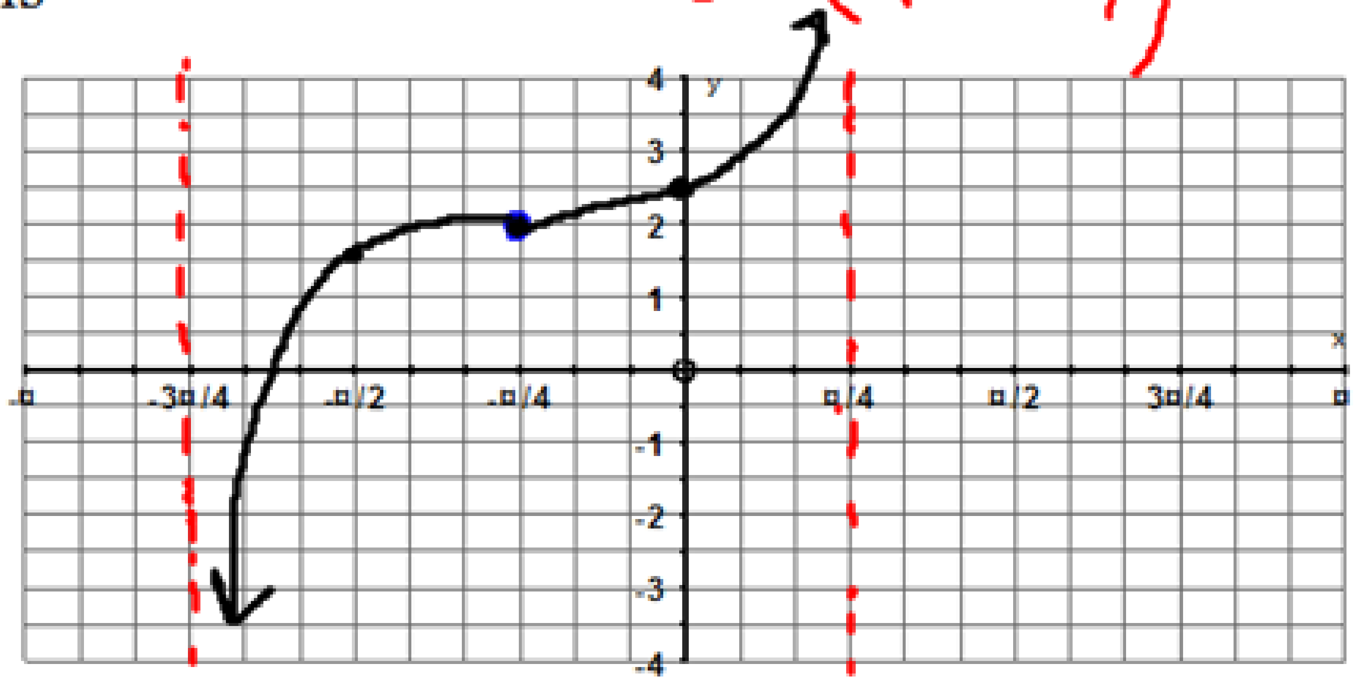
V.A $= -\frac{\pi}{4} \pm \frac{\pi}{2} = \left(\frac{\pi}{4}, -\frac{3\pi}{4}\right) \pm \pi$

List transformations

VS $= 1/2$

VT $= 2$

HT $= -\pi/4$



Period:

HS $\cdot\, \pi$

π

Domain:

$\{x \mid x \neq \frac{\pi}{4} \pm \pi k, k \in I, x \in R\}$

Range:

$\{y \mid y \in R\}$

$y = -2\tan\left(\left(x + \frac{\pi}{12}\right)\right) + 1$

List transformations

VS 2

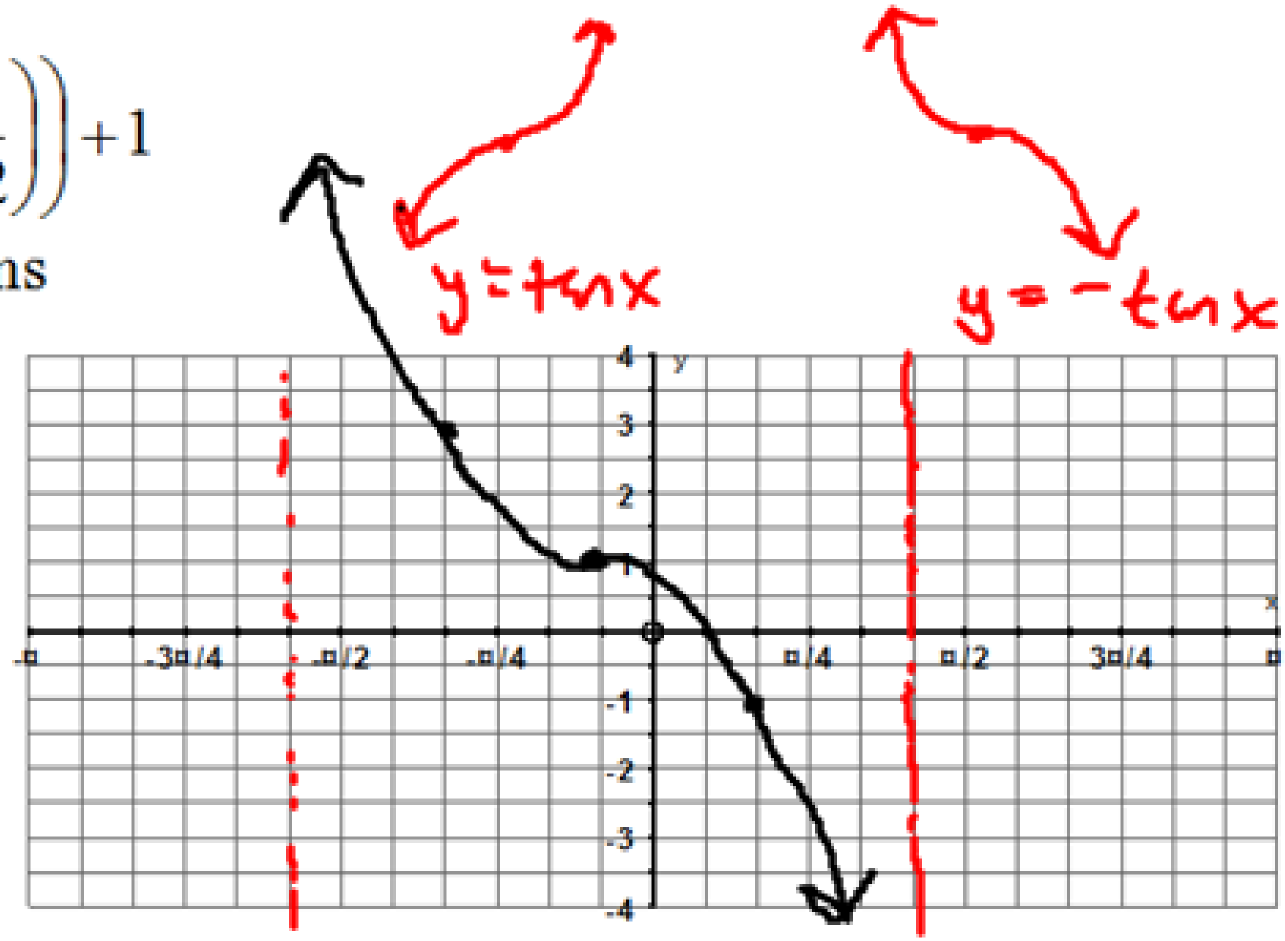
VT = 1

HT = $-\pi/12$

Period:

π

V.A = $\frac{-\pi}{12} \pm \frac{\pi}{2}$



Domain:

$\{x \mid x \neq \frac{5\pi}{12} \pm \pi k, k \in I, x \in R\}$

Range:

$\{y \mid y \in R\}$

HW: WS

and pg 262 # 1–3, 5, 7, 8, 10, 12