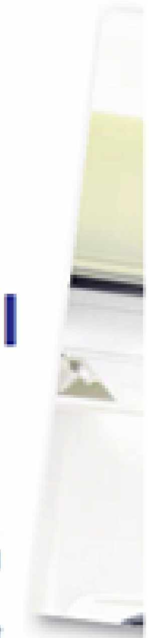
6. A) Consider the function $f(x) = 2(x-1)^2 - 3$. Determine the equation of each function. [3 pts]

Rx Ry

i) $-f(x)$	ii) $f(-x)$	iii) $-f(-x)$
$-f(x) = -\left[2(x-1)^2 - 3\right]$	$f(-x) = 2(-x-1)^2 - 3$	$-f(-x) = -\left[2(x+1)^2 - 3\right]$
$-f(x) = -2(x-1)^2 + 3$	$f(-x) = 2(-1(x+1))^2 - 3$	$-f(-x) = -2(x+1)^2 + 3$
	$f(-x) = 2(x+1)^2 - 3$	

3.4

Equations and Graphs of Polynomial Functions



Focus on…

- describing the relationship between zeros, roots, and x-intercepts of polynomial functions and equations
- sketching the graph of a polynomial function without technology
- modelling and solving problems involving polynomial functions

Difference between **roots**, ***x*-intercepts**, and **zeros**:

Find the **roots** of the equation $3x^3 - 10x^2 - 23x - 10 = 0$

Find the **zeros** of the function $f(x) = 3x^3 - 10x^2 - 23x - 10$

Find ***x*-intercepts** of the graph of $f(x) = 3x^3 - 10x^2 - 23x - 10$

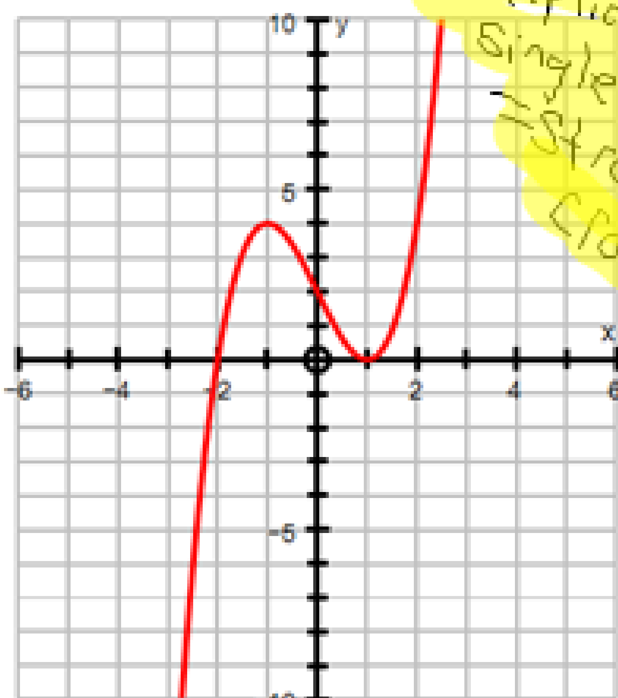
Repeated Roots:

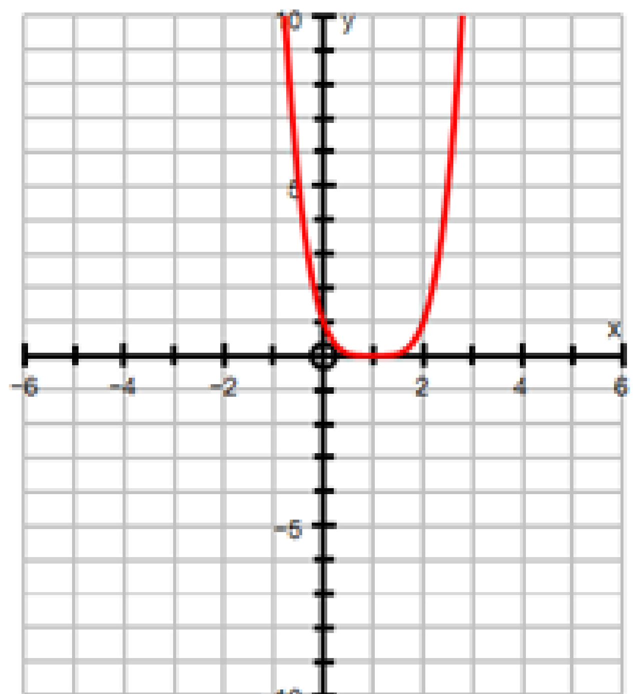
If a polynomial has a factor $x - a$ that is repeated n times, then $x = a$ is a zero of **multiplicity**, n.

Example:

$$f(x) = \overbrace{(x-1)(x-1)}^{\text{multiplicity 2 (double root)}}\underbrace{(x+2)}_{\text{multiplicity 1 | single root = straight cross}}$$

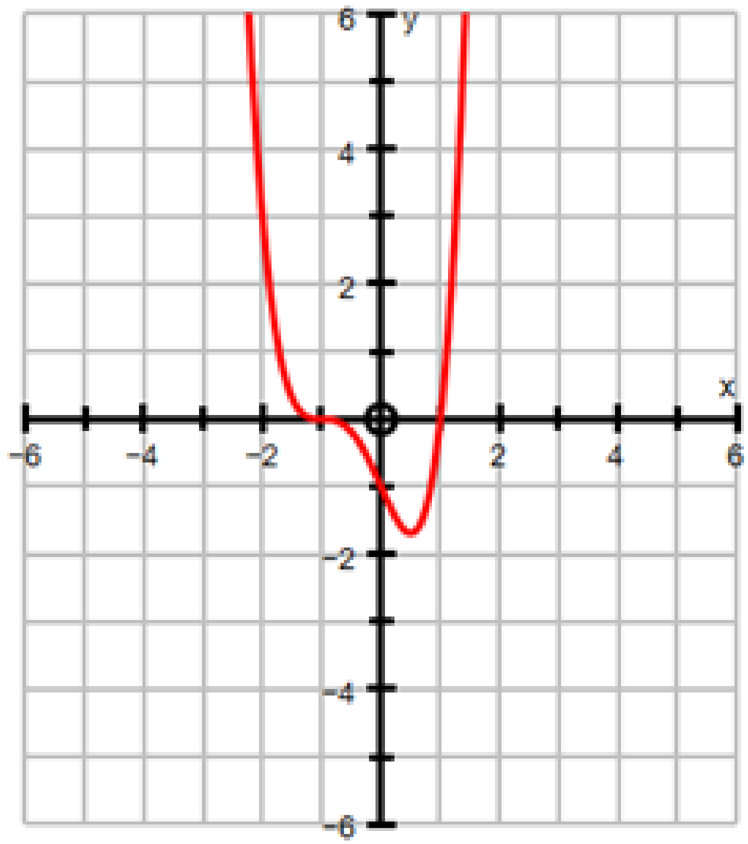
$$f(x) = (x-1)^4 \rightarrow \text{multiplicity 4 (even) } \underline{\text{bounce}}$$



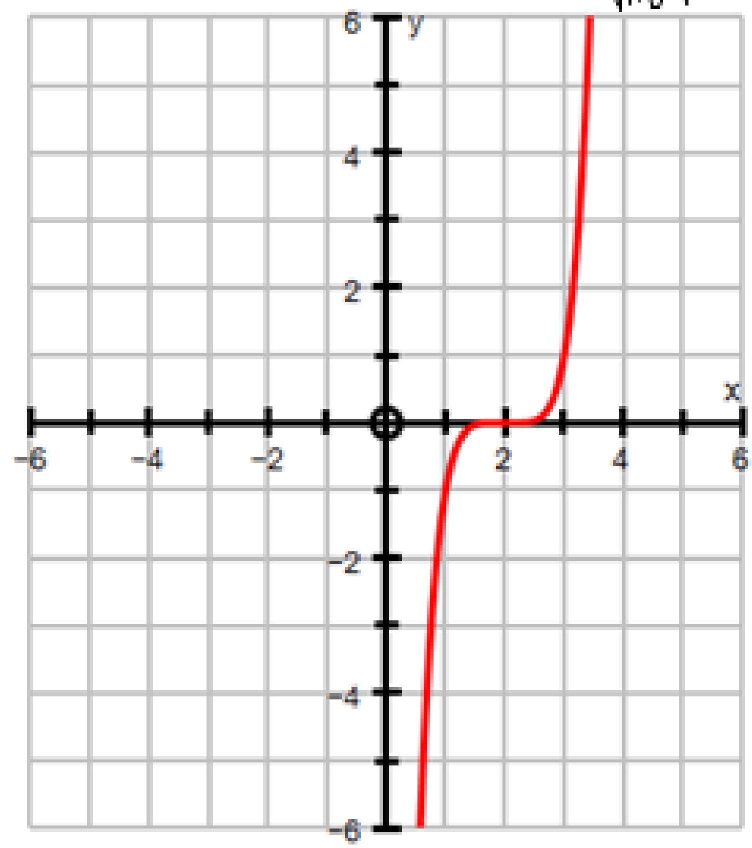


If the multiplicity of the zero is even, then the vertex touches the x-axis at $x = a$ (bounces)

$f(x) = \overbrace{(x+1)(x+1)(x+1)}^{\text{multiplicity 3}}(x-1)$



$f(x) = (x-2)^5 \rightarrow$ odd multiplicity



and greater than 1

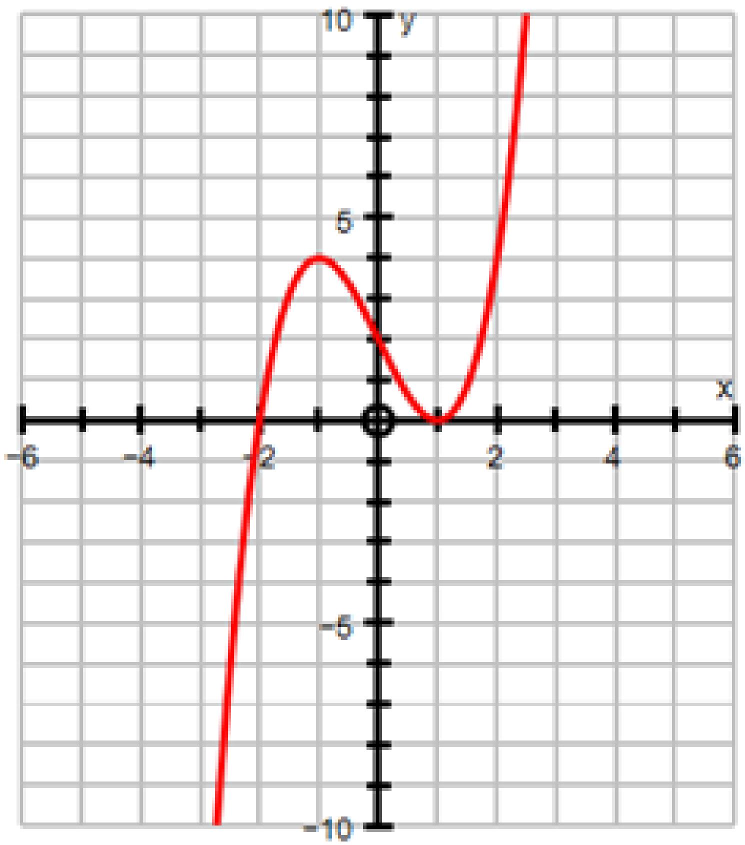
If the multiplicity of the zero is odd, then the function passes through the x-axis at $x = a$ (with a twist).

Analysing Graphs of Polynomial Functions:

- Identify the x and y-intercepts. Check for multiple roots.
- Determine the minimum degree of the polynomial.
- Use the end behaviour to determine the sign of the leading coefficient and to see if the polynomial is even or odd.
- Identify the intervals where the function is positive and where it is negative.

* neither + or − on x axis

y int (0, 2)



x ints $\underbrace{(-2,0)}_{\text{single root}}$ $\underbrace{(1,0)}_{\text{double root}}$

min degree = 3

LC = +

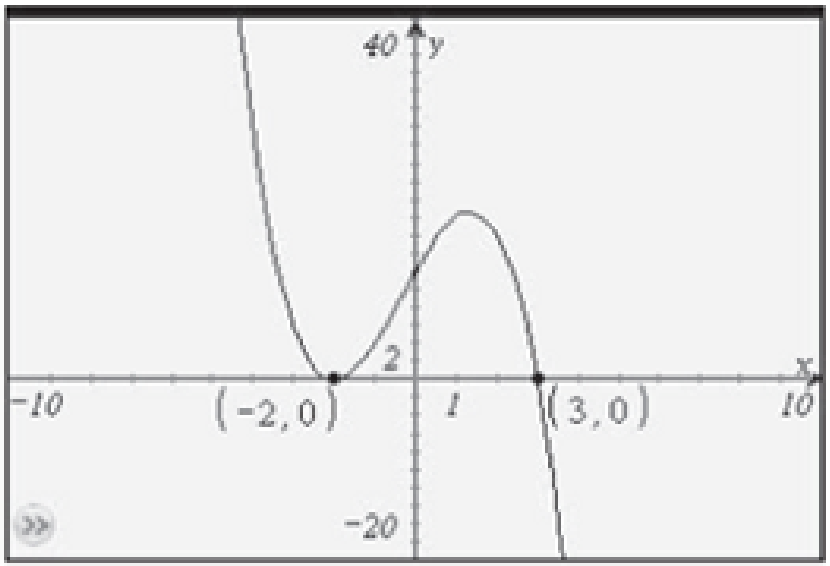
Negative: $x \in (-\infty, -2)$

Positive: $x \in (-2, 1) \cup (1, \infty)$

Pg. 139

Your Turn

For the graph of the polynomial function shown, determine
- the least possible degree 3
- the sign of the leading coefficient −
- the x-intercepts and the factors of the function of least possible degree
- the intervals where the function is positive and the intervals where it is negative



x ints $(-2,0)$ $(3,0)$

Factors $(x+2)^2$ $(x-3)$

P: $x \in (-\infty, -2) \cup (-2, 3)$

N: $x \in (3, \infty)$

Sketching Graphs of Polynomial Functions
(without technology)

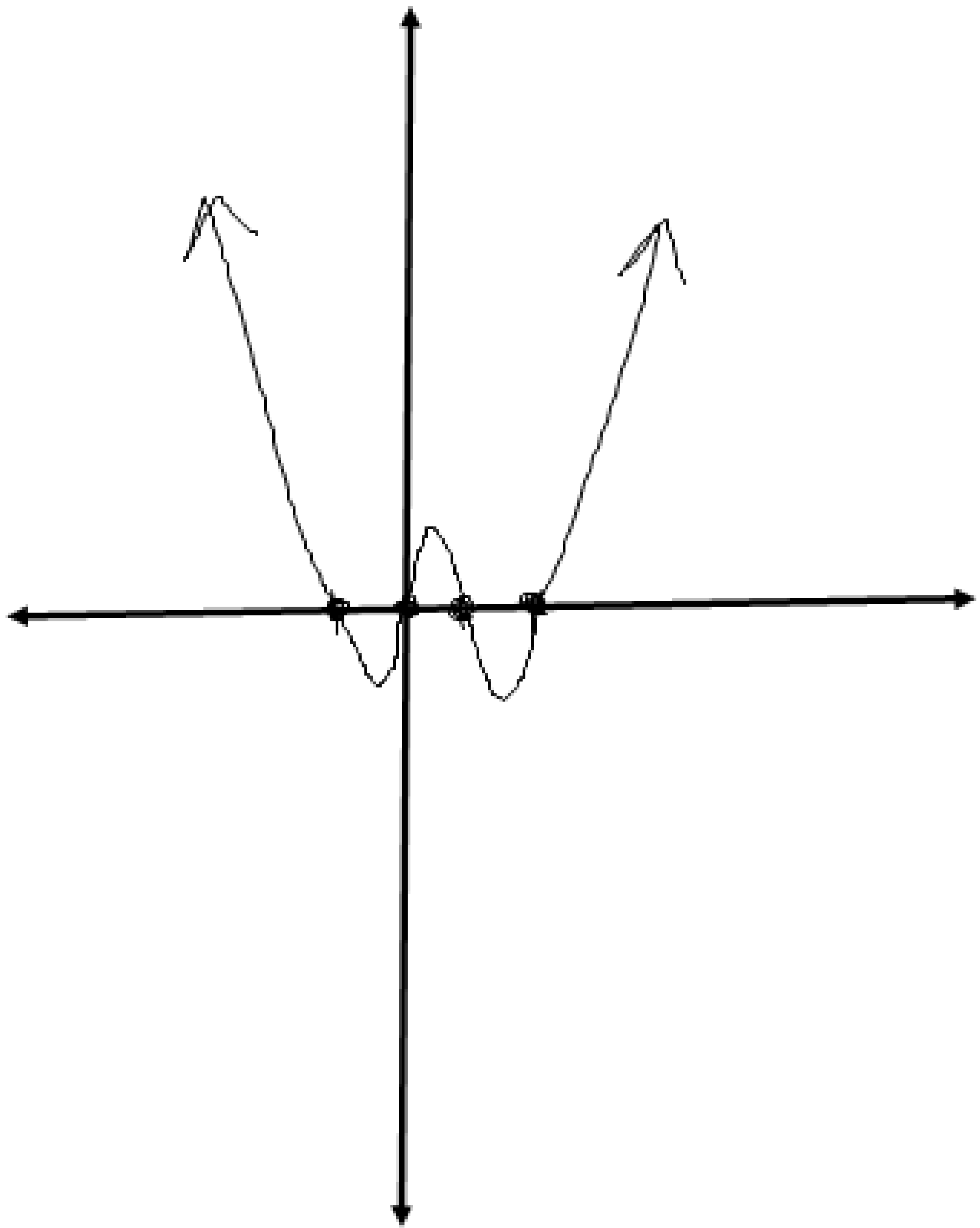
To sketch a graph you need to identify:

1. The degree of the function
2. The leading coefficient
3. The end behaviour
4. The zeros/x-int.\
5. The y-int.
6. The intervals where the function is positive or negative.

Examples: Sketch the graphs of each polynomial function:

1. $f(x) = x(x-1)(x+1)(x-2)$

Degree	4
Leading Coefficient	+1
End Behaviour	starts high ends high
Zeros/x-intercepts	(1,0) cross (−1,0) cross (2,0) cross (0,0) cross
y-intercept	(0,0)
Interval(s) where the function is positive or negative	P: $x \in (-\infty, -1) \cup (0,1) \cup (2, \infty)$ N: $x \in (-1, 0) \cup (1, 2)$



2. $f(x) = 4 - 3x^2 - x^3$

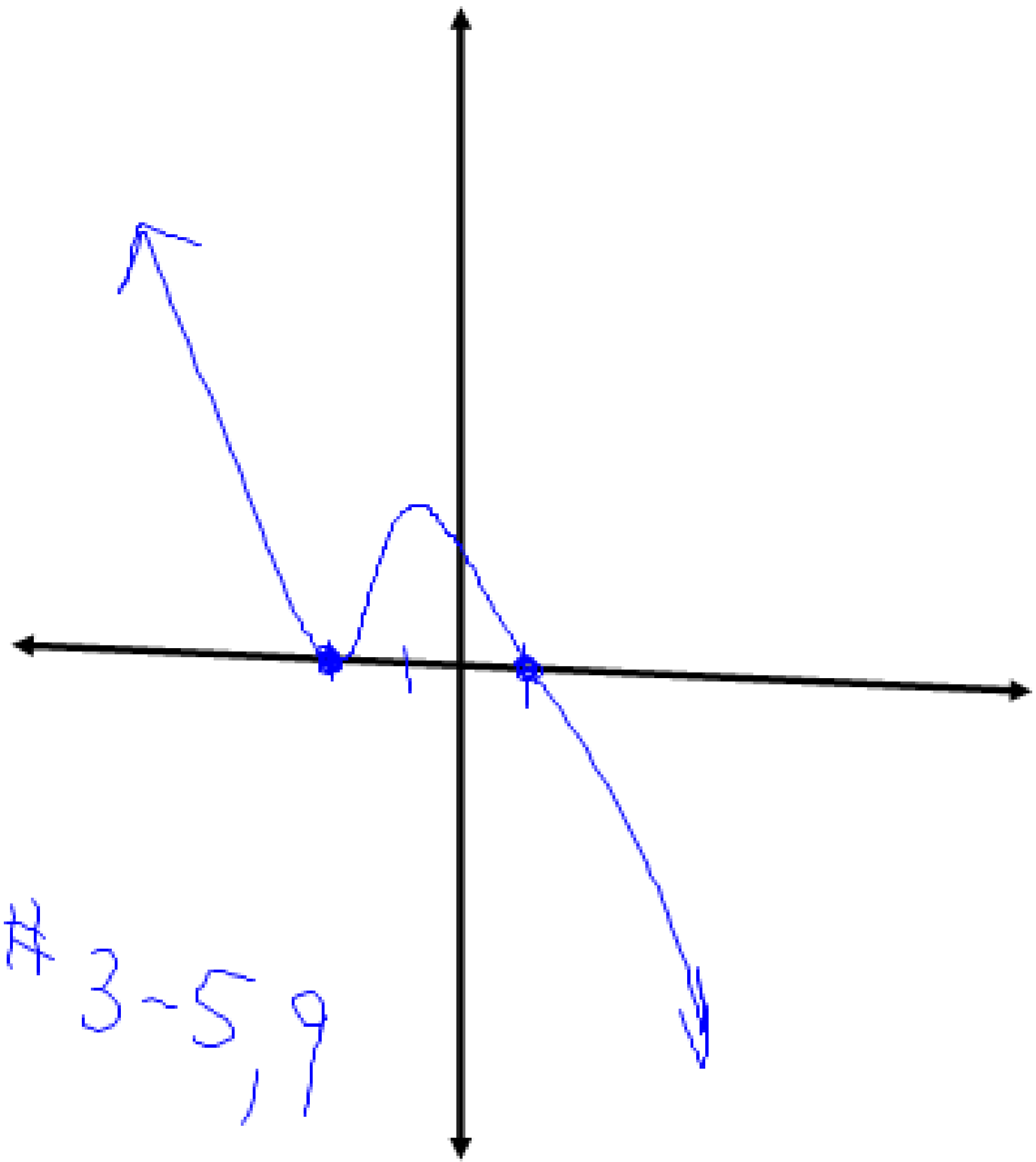
$F(x) = -x^3 - 3x^2 + 4$

$f(1) = -(1)^3 - 3(1)^2 + 4$

$= 0$

$(x-1)$ factor

Degree 3	$1 \,\big|\, -1 \quad -3 \quad 0 \quad 4$ $\quad\; -1 \quad -4 \quad -4$ $-1 \quad -4 \quad -4 \,\big|\, 0$
Leading Coefficient $-$	$-x^2 - 4x - 4$
End Behaviour starts high ends low	$-1(x^2 + 4x + 4)$ $-1(x-1)(x+2)(x+2)$
Zeros/x-intercepts $(1, 0)$ cross $(-2, 0)$ bounce	
y-intercept $(0, 4)$	
Interval(s) where the function is positive or negative	



HW:
Pg 148 # 3-5, 9

3. $f(x) = -x^4 + 4x^3 - x^2 - 6x$

H/W pg 148 #3-5, 9

Degree	
Leading Coefficient	
End Behaviour	
Zeros/x-intercepts	
y-intercept	
Interval(s) where the function is positive or negative	

4. $f(x) = x^5 + 2x^4 - 6x^3 - 20x^2 - 19x - 6$

Degree	
Leading Coefficient	
End Behaviour	
Zeros/x-intercepts	
y-intercept	
Interval(s) where the function is positive or negative	