Date: Block: Name: #

Factoring

1. Factor fully using the rational root theorem, or one of the special factoring techniques discussed in class.

a) $x^3 - 4x^2 + x + 6$

b) $2x^3 + x^2 - 13x + 6$

c) $x^3 + x^2 - 16x + 20$

d) $(x-5)(3x^3 - x^2 - 20x - 12)$

e) $x^3 - 4x^2 - 4x + 16$

f) $x^3 - 64$

g) $5a^4 - 135a$

h) $x^3 - 5x^2 - 9x + 45$

i) $x^4 - 13x^2 + 36$

j) $x^4 + 8x^3 - 2x^2 - 16x$

k) $4x^3 - 8x^2 - 25x + 50$

l) $36x^2 - x^4 - 100$

m) $x^4 - x^3 - x + 1$

n) $x^5 - 2x^4 - x^3 + 2x^2 - 12x + 24$

Factoring Review

Need to brush up on your factoring?!?!

2. You already know three factoring methods. They are: a) common factoring, b) difference of squares factoring and c) trinomial factoring. Use which ever method is appropriate to completely factor the following polynomial expressions.

a) $x^2 - 9x + 18$

b) $x^2 - 64$

c) $15x^3d - 10x^2d - 25xd$

d) $3x^2 - 33x + 72$

e) $2x^2 - 11x + 5$

f) $36x^2 - 121$

g) $16x^2 - 25y^2$

h) $50x^3 - 72xy^2$

i) $12x^2 - 17x + 6$

j) $9x^2 + 6x - 8$

k) $x^4 - 81$

l) $6x^2 + xy - 12y^2$

3. Factor the following using Factor by grouping.

a) $5x^3 - 3x^2 + 10x - 6$

b) $2ax - bx + 2ay - by$

c) $3x^5 - x^4 - 9x + 3$

d) $c^2 - ck - cx + kx$

e) $m^2 - m + mn - n$

f) $x^3 + 7x^2 - 5x - 35$

g) $ax + by + bx + ay$

h) $4mx + 2ny - 8my - nx$

i) $x^2y - y^2 - x^3 + xy$

j) $mx + 2y + my + 2x$

k) $10x^2 + 3y - 5xy - 6x$

l) $a^2b^2 - 7ba^2 + 13a^2 - 4b^2 + 28b - 52$

m) $ax - 3a + 3b - bx$

n) $3ab - 20cd - 15ac + 4bd$

o) $ax^2 - bx^2 + ax - bx + a - b$

4. Grouping of terms is also used to create differences of squares.

a) $c^2 + 6c + 9 - d^2$

b) $a^2 + 4a + 4 - 9x^2$

c) $b^2 - a^2 + 2ac - c^2$

d) $k^2 - y^2 - 2y - 1$

e) $b^2 - 2ab + a^2 - 1$

f) $x^2 - 9y^2 + 6y - 1$

Factoring

1 A) $x^3 - 4x^2 + x + 6$

$x = -1$ is a root

$$\begin{array}{r|cccc} -1 & 1 & -4 & 1 & 6 \\ & \downarrow & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$(x+1)(x^2-5x+6)$

$(x+1)(x-3)(x-2)$

B) $2x^3 + x^2 - 13x + 6$

$x = 2$ is a root

$$\begin{array}{r|cccc} 2 & 2 & 1 & -13 & +6 \\ & \downarrow & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$(x-2)(2x^2+5x-3)$

$(x-2)(2x-1)(x+3)$

c) $x^3 + x^2 - 16x + 20$

$x = 2$ is a root

$$\begin{array}{r|cccc} 2 & 1 & 1 & -16 & 20 \\ & \downarrow & 2 & 6 & -20 \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

$(x-2)(x^2+3x-10)$

$(x-2)(x+5)(x-2)$

d) $(x-5)(3x^3 - x^2 - 20x - 12)$

$x = -2$ is a root

$$\begin{array}{r|cccc} -2 & 3 & -1 & -20 & -12 \\ & \downarrow & -6 & 14 & 12 \\ \hline & 3 & -7 & -6 & 0 \end{array}$$

$(x-5)(x+2)(3x^2-7x-6)$

$(x-5)(x+2)(3x+2)(x-3)$

e) $x^3 - 4x^2 - 4x + 16$

$x^2(x-4) - 4(x-4)$

$(x-4)(x^2-4)$

$(x-4)(x-2)(x+2)$

f) $x^3 - 64$

$(x)^3 - (4)^3$

$(x-4)(x^2+4x+16)$

g) $5a^4 - 135a$

$5a(a^3-27)$

$5a(a-3)(a^2+3a+9)$

h) $x^3 - 5x^2 - 9x + 45$

$x^2(x-5) - 9(x-5)$

$(x-5)(x^2-9)$

$(x-5)(x+3)(x-3)$

i) $x^4 - 13x^2 + 36$

$(x^2-9)(x^2-4)$

$(x+3)(x-3)(x-2)(x+2)$

j) $x^4 + 8x^3 - 2x^2 - 16x$

$x(x^3 + 8x^2 - 2x - 16)$

$x(x^2(x+8) - 2(x+8))$

$x(x+8)(x^2-2)$ ← Oh no! what to do?!?

$x(x+8)(x-\sqrt{2})(x+\sqrt{2})$

k) $4x^3 - 8x^2 - 25x + 50$

$4x^2(x-2) - 25(x-2)$

$(x-2)(4x^2-25)$

$(x-2)(2x-5)(2x+5)$

tricky...

l) $36x^2 - x^4 - 100$

$-x^4 + 36x^2 - 100$

$-(x^4 - 36x^2 + 100)$

$-(x^4 - 20x^2 + 100 - 16x^2)$

diff of squares →

$-((x^2-10)^2 - 16x^2)$

$-(x^2 - 10 + 4x)(x^2 - 10 - 4x)$

$-(x^2 + 4x - 10)(x^2 - 4x - 10)$

yucky... x-int will be irrational....

m) $x^4 - x^3 - x + 1$

$x^3(x-1) - 1(x-1)$

$(x-1)(x^3-1)$

$(x-1)(x-1)(x^2+x+1)$

n) $\underline{x^5 - 2x^4} - \underline{x^3 + 2x^2} - \underline{12x + 24}$

$x^4(x-2) - x^2(x-2) - 12(x-2)$

$(x-2)(x^4 - x^2 - 12)$

$(x-2)(x^2-4)(x^2+3)$

$(x-2)(x-2)(x+2)(x^2+3)$

Factoring Review

2 A) $x^2-9x+18$
$(x-6)(x-3)$

B) x^2-64
$(x-8)(x+8)$

C) $15x^3d-10x^2d-25xd$
$5xd(3x^2-2x-5)$
$5xd(3x^2-5x+3x-5)$
$5xd(x(3x-5)+1(3x-5))$
$5xd(3x-5)(x+1)$

d) $3x^2-33x+72$
$3(x^2-11x+24)$
$3(x-8)(x-3)$

e) $2x^2-11x+5$
$(2x-1)(x-5)$

f) $36x^2-121$
$(6x+11)(6x-11)$

g) $16x^2-25y^2$
$(4x-5y)(4x+5y)$

h) $50x^3-72xy^2$
$2x(25x^2-36y^2)$
$2x(5x-6y)(5x+6y)$

i) $12x^2-17x+6$
$12x^2-9x-8x+6$
$3x(4x-3)-2(4x-3)$
$(4x-3)(3x-2)$

j) $9x^2+6x-8$
$9x^2+12x-6x-8$
$3x(3x+4)-2(3x+4)$
$(3x+4)(3x-2)$

k) x^4-81
$(x^2-9)(x^2+9)$
$(x-3)(x+3)(x^2+9)$

l) $6x^2+xy-12y^2$
$6x^2+9xy-8xy-12y^2$
$3x(2x+3y)-4y(2x+3y)$
$(2x+3y)(3x-4y)$

3) A) $5x^3 - 3x^2 + 10x - 6$
$x^2(5x-3) + 2(5x-3)$
$(5x-3)(x^2+2)$

B) $2ax - bx + 2ay - by$
$x(2a-b) + y(2a-b)$
$(2a-b)(x+y)$

C) $3x^5 - x^4 - 9x + 3$
$x^4(3x-1) - 3(3x-1)$
$(3x-1)(x^4-3)$

D) $c^2 - ck - cx + kx$
$c(c-k) - x(c-k)$
$(c-k)(c-x)$

E) $m^2 - m + mn - n$
$m(m-1) + n(m-1)$
$(m-1)(m+n)$

F) $x^3 + 7x^2 - 5x - 35$
$x^2(x+7) - 5(x+7)$
$(x+7)(x^2-5)$

G) $ax + by + bx + ay$
$ax + ay + bx + by$
$a(x+y) + b(x+y)$
$(a+b)(x+y)$

H) $4mx + 2ny - 8my - nx$
$4mx - 8my - nx + 2ny$
$4m(x-2y) - n(x-2y)$
$(x-2y)(4m-n)$

I) $x^2y - y^2 - x^3 + xy$
$x^2y - x^3 - y^2 + xy$
$x^2(y-x) - y(y-x)$
$(y-x)(x^2-y)$

J) $mx + 2y + my + 2x$
$mx + my + 2x + 2y$
$m(x+y) + 2(x+y)$
$(x+y)(m+2)$

K) $10x^2 + 3y - 5xy - 6x$
$10x^2 - 6x - 5xy + 3y$
$2x(5x-3) - y(5x-3)$
$(5x-3)(2x-y)$

L) $a^2b^2 - 7ba^2 + 13a^2 - 4b^2 + 28b - 52$
$a^2(b^2 - 7b + 13) - 4(b^2 - 7b + 13)$
$(a^2-4)(b^2-7b+13)$
$(a-2)(a+2)(b^2-7b+13)$

M) $ax - 3a + 3b - bx$
$a(x-3) - b(x-3)$
$(a-b)(x-3)$

N) $3ab - 20cd - 15ac + 4bd$
$3ab - 15ac + 4bd - 20cd$
$3a(b-5c) + 4d(b-5c)$
$(b-5c)(3a+4d)$

30) $ax^2 - bx^2 + ax - bx + a - b$

$ax^2 + ax + a - bx^2 - bx - b$

$a(x^2 + x + 1) - b(x^2 + x + 1)$

$(x^2 + x + 1)(a - b)$

H a) $c^2 + 6c + 9 - d^2$

$(c + 3)^2 - d^2$

$(c + 3 - d)(c + 3 + d)$

B) $a^2 + 4a + 4 - 9x^2$

$(a + 2)^2 - (3x)^2$

$(a + 2 - 3x)(a + 2 + 3x)$

c) $b^2 - a^2 + 2ac - c^2$

$b^2 - (a^2 - 2ac + c^2)$

$b^2 - (a - c)^2$

$(b - (a - c))(b + (a - c))$

$(b - a + c)(b + a - c)$

d) $k^2 - y^2 - 2y - 1$

$k^2 - (y^2 + 2y + 1)$

$k^2 - (y + 1)^2$

$(k - (y + 1))(k + (y + 1))$

$(k - y - 1)(k + y + 1)$

e) $b^2 - 2ab + a^2 - 1$

$(b - a)^2 - 1$

$(b - a - 1)(b - a + 1)$

f) $x^2 - 9y^2 + 6y - 1$

$x^2 - (9y^2 - 6y + 1)$

$x^2 - (3y - 1)^2$

$(x - (3y - 1))(x + (3y - 1))$

$(x - 3y + 1)(x + 3y - 1)$