4 Find a given that:

a P(2, 3) and Q(a, -1) are 4 units apart

$x_1\ y_1$ $x_2\ y_2$ distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$4 = \sqrt{(a-2)^2 + (-1-3)^2}$$

$$(4)^2 = \left(\sqrt{(a-2)^2 + 16}\right)^2$$

$$16 = (a-2)^2 + 16$$

-16 -16

$$\sqrt{0} = \sqrt{(a-2)^2}$$

$$0 = a - 2$$

$$\boxed{2 = a}$$

d A(0, a) is equidistant from P(3, −3) and Q(−2, 2).

$x_2 y_2$ $x_1 y_1$ $x_1 y_1$
$x_2 y_2$ same distance

$d_{AP} = d_{AQ}$

$$\left(\sqrt{(0-3)^2+(a+3)^2}\right)^2 = \left(\sqrt{(0+2)^2+(a-2)^2}\right)^2$$

$$(0-3)^2+(a+3)^2 = (0+2)^2+(a-2)^2$$

$$9+(a+3)^2 = 4+(a-2)^2$$

$$9+a^2+6a+9 = 4+a^2-4a+4$$

$-a^2$ $-a^2$

$$9+6a+9 = 4-4a+4$$

$+4a$ $+4a$

$$18+10a = 8$$

-18 -18

$$10a = -10$$

10 10

$$a = -1$$

5c – Gradient (Slope)

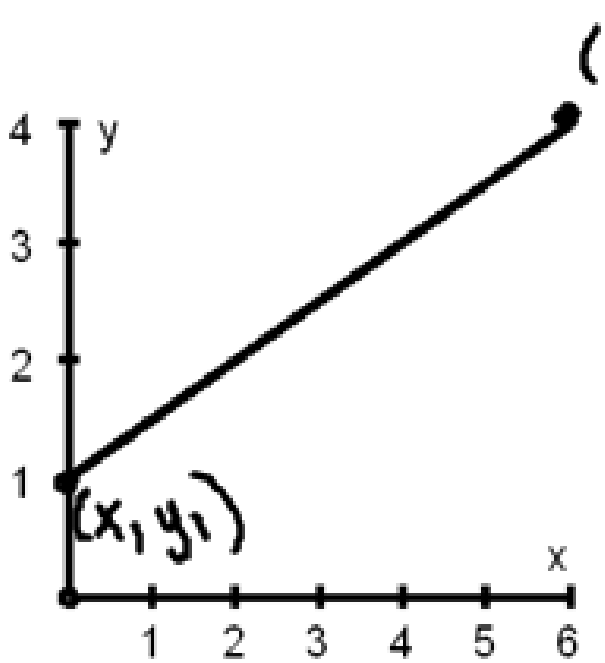
M = midpoint m = slope

The gradient or slope of a line is a measure of its steepness.

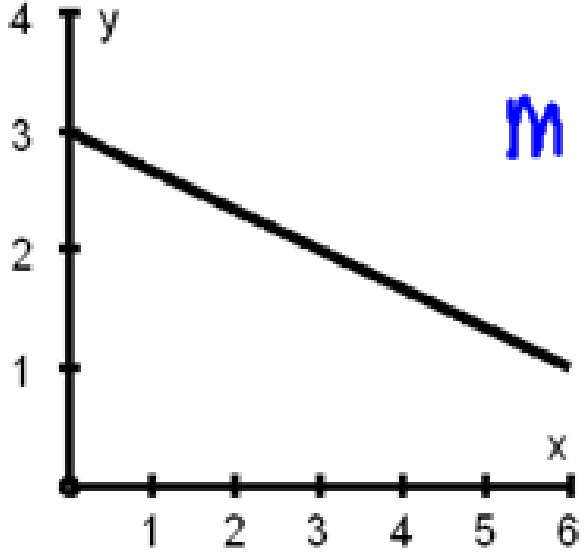
slope

$$gradient = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\updownarrow}{\leftrightarrow}$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$$

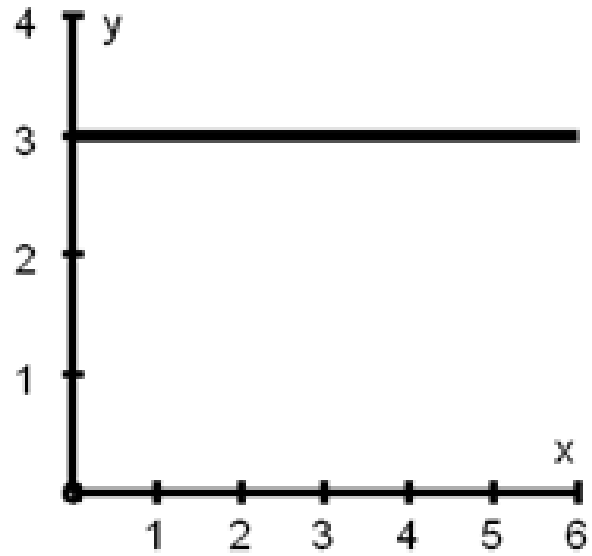


Positive Slope



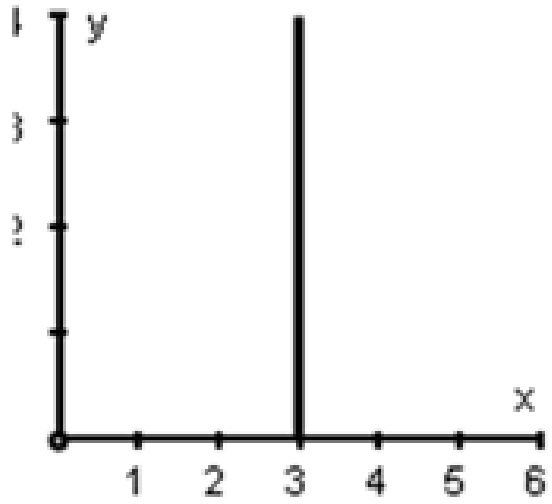
Negative Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



$$m = \frac{\Delta y}{\Delta x} = \frac{0}{6} = 0$$

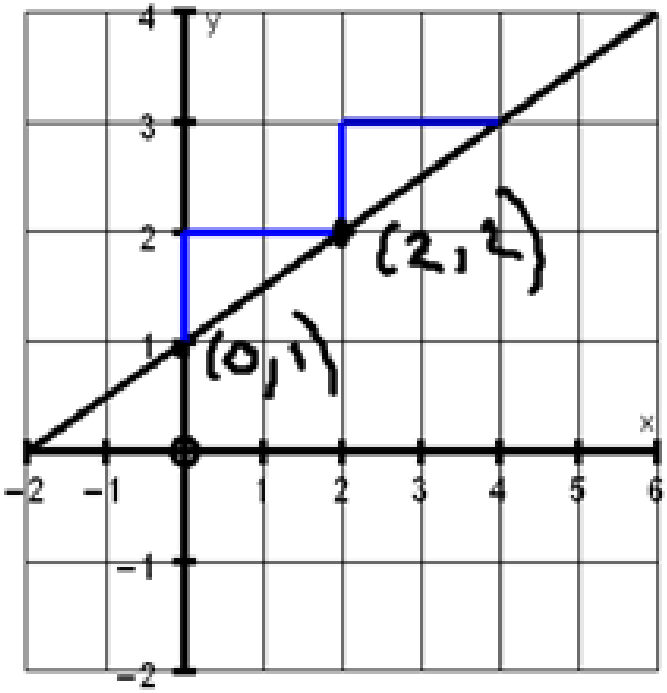
zero slope



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{3-3} = \frac{4}{0}$$

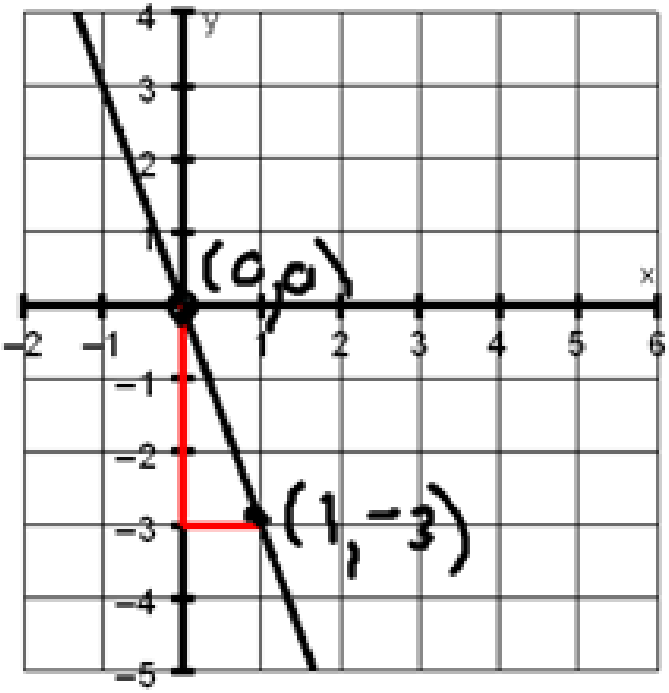
undefined slope

Example: Find the gradient of each line:



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{2-0} = \frac{1}{2}$$

$\frac{1}{2}$



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-0}{1-0} = \frac{-3}{1}$$

-3

Example: Find the gradient of the line segment joining the following points.

A) (2, -1) and (-3, -7)

$m = \frac{6}{5}$

B) (1, 1) and (4, 5)

$m = \frac{4}{3}$

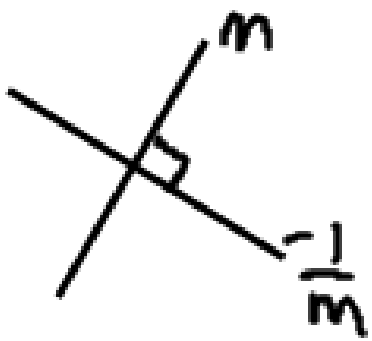
Parallel Lines-

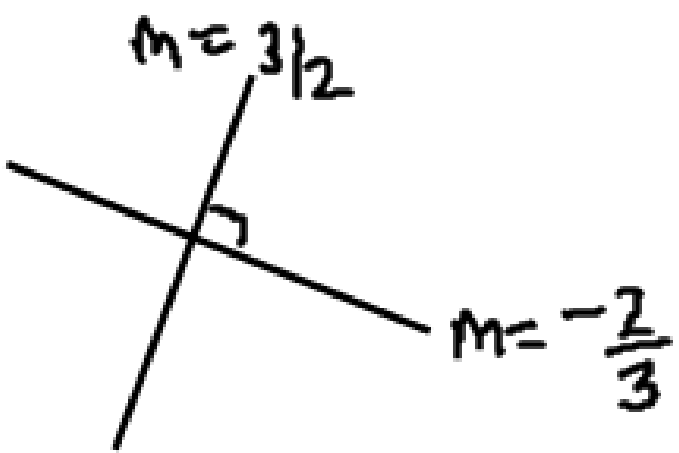
- If two lines are parallel, then they have equal gradient.
- If two lines have equal gradient, then they are parallel.

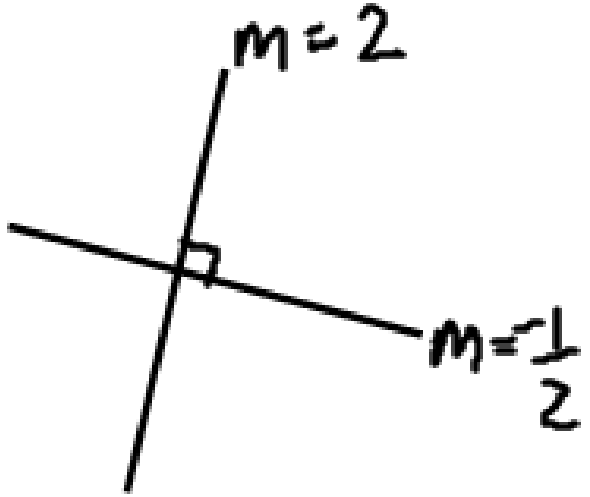
Perpendicular Lines- (meet at a 90° angle)

- If the lines are perpendicular, then their gradients have negative reciprocals.

$$m_2 = \frac{-1}{m_1}$$







** Flip it and change the sign.

Examples:

1. A line has a gradient 4. Find the gradient of:

(a) all lines parallel to the line. 4

(b) all lines perpendicular to the line. $-\frac{1}{4}$

2. A line has a gradient $-\frac{5}{3}$. Find the gradient of:

(a) all lines parallel to the line. $-\frac{5}{3}$

(b) all lines perpendicular to the line. $\frac{3}{5}$

3. Find a given that the line joining M$(3,a)$ to N$(a,5)$ is parallel to a line with gradient $-\frac{2}{5}$.

$x_1 y_1 \quad x_2 y_2$

MN has a slope of $-2/5$

$$m = \frac{5-a}{a-3}$$

$$\frac{-2}{5} = \frac{(5-a)}{a-3} \cdot 5$$

$$(a-3)2 = 25 - 5a \quad (a \neq 3)$$

$$(a-3)(-2) = 25-5a$$

$$-2a + 6 = 25 - 5a$$

$$+5a \qquad +5a$$

$$3a + 6 = 25$$

$$-6 \quad -6$$

$$3a = 19$$

$$a = 19/3$$

Collinear Points –

- Three or more points are collinear if they lie on the same straight line

Example: Determine whether or not the following sets of three points are collinear. A(0,-2), B(-1,-5), and C(3, 7)