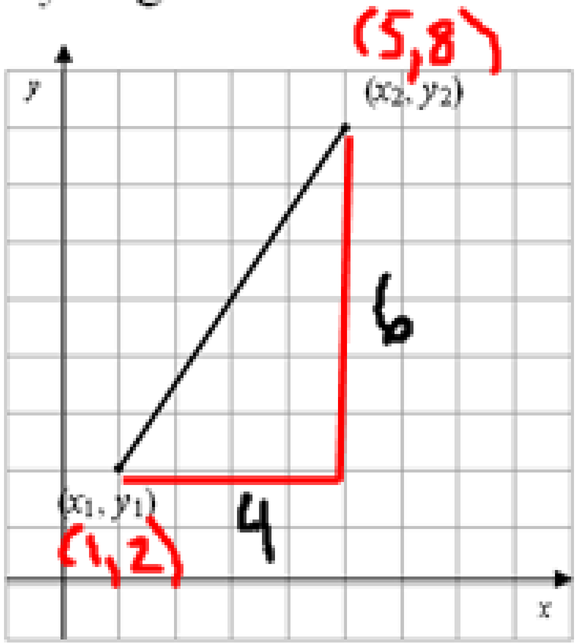
CH 5 A - Length of a Line Segment #Distance

Given the coordinates of the endpoints of a line segment, we can use Pythagorean Theorem to find the length of the segment:



$$y_2 - y_1 = 6 \qquad d^2 = 4^2 + 6^2$$

$$x_2 - x_1 = 4 \qquad d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d^2 = (5-1)^2 + (8-2)^2$$

$$d^2 = 4^2 + 6^2$$

$$d^2 = 52$$

$$d = \pm\sqrt{52} \qquad d > 0$$

$$\boxed{d = \sqrt{52}}$$

FORMULA: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: Calculate the length of the line segment joining:

$x_1 y_1$ $x_2 y_2$

J(5, 1) and K(-3, -8)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-3-5)^2 + (-8-1)^2}$$

$$d = \sqrt{(-8)^2 + (-9)^2}$$

$$d = \sqrt{64 + 81}$$

$d = \sqrt{145}$ exact radical answer

Example: Use the distance formula to show that the following triangle is a right angled triangle. yes

A (3, -4), B (-2, -5), C (2, 1)

Step 1- Draw a rough sketch

Step 2- Find the length of each line segment

AC

A $(3, -4)$
$x_1 \; y_1$

C $(2, 1)$
$x_2 \; y_2$

$d = \sqrt{(2-3)^2 + (1 - -4)^2}$

$d = \sqrt{1 + 25}$

$d = \sqrt{26}$

AB

A $(3, -4)$
$x_1 \; y_1$

B $(-2, -5)$
$x_2 \; y_2$

$d = \sqrt{(-2-3)^2 + (-5 - -4)^2}$

$d = \sqrt{25 + 1}$

$d = \sqrt{26}$

BC

B $(-2, -5)$
$x_1 \; y_1$

C $(2, 1)$
$x_2 \; y_2$

$d = \sqrt{(2 - -2)^2 + (1 - -5)^2}$

$d = \sqrt{16 + 36}$

$d = \sqrt{52}$

longest

c

$a^2 + b^2 = c^2$

$\sqrt{26}^2 + \sqrt{26}^2 = \sqrt{52}^2$

$52 = 52$ ✓

Example: Given A(-5, 4), B(2, 0), and C(-1, -3), prove that ΔABC is an isosceles triangle.

$\underline{AB}$

$A(-5, 4)$ — $x_1\ y_1$

$B(2, 0)$ — $x_2\ y_2$

$d=\sqrt{(2--5)^2+(0-4)^2}$

$d=\sqrt{49+16}$

$d=\sqrt{65}$

$\underline{BC}$

$B(2, 0)$ — $x_1\ y_1$

$C(-1, -3)$ — $x_2\ y_2$

$d=\sqrt{(-1-2)^2+(-3-0)^2}$

$d=\sqrt{9+9}$

$d=\sqrt{18}$

$\underline{AC}$

$A(-5, 4)$ — $x_1\ y_1$

$C(-1, -3)$ — $x_2\ y_2$

$d=\sqrt{(-1--5)^2+(-3-4)^2}$

$d=\sqrt{16+49}$

$d=\sqrt{65}$

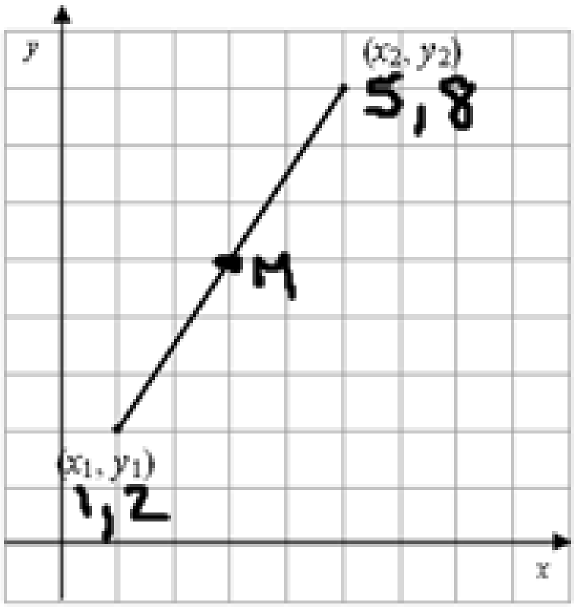
Two sides equal length means isosceles.

CH 5 B - Midpoint of a Line Segment

The midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) is the average of the endpoints:

$$M = \left(\frac{5+1}{2}, \frac{8+2}{2}\right)$$

$$M = (3, 5)$$



Midpoint represented with capital M

FORMULA: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Example: Calculate the midpoint of the segment joining the points:

A) P(-6, 5) and Q(3, 7)

$$M = \left(\frac{-6+3}{2}, \frac{5+7}{2}\right)$$

$$M = \left(-3/2, 6\right)$$

$$M = \left(-1.5, 6\right)$$

B) J(13, -1) and K(-7, -8)

$$M = \left(\frac{13+-7}{2}, \frac{-1+-8}{2}\right)$$

$$M = \left(3, -9/2\right)$$

$$M = \left(3, -4.5\right)$$

Example: M is the midpoint of [AB]. Find the coordinates of A if B is (5,-2) and $\underbrace{\text{M is (2,1)}}_{\text{Midpoint}}$

$x_1 y_1$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$(2,1) = \left(\frac{5 + x_2}{2}, \frac{-2 + y_2}{2} \right)$$

$$2 = \frac{5 + x_2}{2} \qquad 1 = \frac{-2 + y_2}{2}$$

$$4 = 5 + x_2 \qquad 2 = -2 + y_2$$

$$-1 = x_2 \qquad 4 = y_2$$

$A(x_2, y_2)$

$A(-1, 4)$

HW: Pg 107 #1-4
Pg 109 #1-4