1. Write the equation of the line (in gradient-intercept form) with the following conditions:

(a) passing through the points (4,7) and (-2,-1)

$m = \frac{-1-7}{-2-4} = \frac{-8}{-6} = \frac{4}{3}$

$\frac{4}{3} = \frac{y-7}{x-4}$

$4x - 16 = 3y - 21$

$3y = 4x + 5$

$y = \frac{4}{3}x + \frac{5}{3}$

(b) passing through (-5,2) and parallel to the line $2y - 5x = 10$.

$2y = 5x + 10$

$y = \frac{5}{2}x + 5$

$m = \frac{5}{2}$

$\frac{y-2}{x-5} = \frac{5}{2}$

$2y - 4 = 5x + 25$

$2y = 5x + 29$

$y = \frac{5}{2}x + \frac{29}{2}$

(c) y – intercept of 7 and perpendicular to the line $x - 3y = 4$.

$-3y = -x + 4$

$y = \frac{1}{3}x - \frac{4}{3}$

$y = -3x + 7$

(d) passing through (1,4) and perpendicular to the line $x - 2y = -5$.

$-2y = -x - 5$

$y = \frac{1}{2}x + \frac{5}{2}$

$m = -2$

$-2 = \frac{y-4}{x-1}$

$-2x + 2 = y - 4$

$y = -2x + 6$

2. Find the equation (in general form) of the perpendicular bisector of AB for:

(a) A (3,-3) and B (1,-1)

$m_{AB} = \frac{-1-(-3)}{1-3} = \frac{2}{-2} = -1$

$M_{AB}\left(\frac{3+1}{2}, \frac{-3+(-1)}{2}\right) = (2,-2)$

Perp. $m = 1$

$1 = \frac{y-(-2)}{x-2}$

$x - 2 = y + 2$

$x - y = 4$

(b) A (1,3) and B (-3,5)

$m_{AB} = \frac{5-3}{-3-1} = \frac{2}{-4} = -\frac{1}{2}$

$M_{AB}\left(\frac{1+(-3)}{2}, \frac{3+5}{2}\right) = (-1, 4)$

Perp. $m = 2$

$2 = \frac{y-4}{x-(-1)}$

$2x + 2 = y - 4$

$2x - y = -6$

3. Two Post offices are located at P (4,5) and Q (2,9) on a Council map. What is the equation of the line which should form the boundary between the two regions being served by the Post Offices?

$m_{PQ} = \frac{9-5}{2-4} = \frac{4}{-2} = -2$

$M_{PQ}\ \left(\frac{4+2}{2}, \frac{5+9}{2}\right)$

$= (3, 7)$

Perp. bis. $m = \frac{1}{2}$

$\frac{y-7}{x-3} = \frac{1}{2}$

$2y - 14 = x - 3$

$-14 + 3 = x - 2y$

$\boxed{x - 2y = -11}$

4. Find the equation (in general form) of the tangent to the circle with centre (5,-4) at the point (8,-2).

$m_{CP} = \frac{-2 - -4}{8-5} = \frac{2}{3}$

tangent: $m = -\frac{3}{2}$ $(8,-2)$

$\frac{y - -2}{x - 8} = -\frac{3}{2}$

$2y + 4 = -3x + 24$

$\boxed{3x + 2y = 20}$

5. Given A (-3,1), B (1,4) and C (4,0):

(a) Show that the triangle ABC is isosceles.

$d_{AB} = \sqrt{(1--3)^2 + (4-1)^2}$

$d_{AB} = \sqrt{16+9} = \sqrt{25}$

$d_{AB} = 5$

$d_{BC} = \sqrt{(4-1)^2 + (0-4)^2}$

$d_{BC} = \sqrt{9+16}$

$d_{BC} = 5$

$d_{AC} = \sqrt{(4--3)^2 + (0-1)^2}$

$d_{AC} = \sqrt{49+1} = \sqrt{50}$

$d_{AC} = 5\sqrt{2}$

Two sides have same length $\therefore$ isosceles

(b) Find the midpoint, X, of AC.

$\left(\frac{-3+4}{2}, \frac{1+0}{2}\right)$

$= \left(\frac{1}{2}, \frac{1}{2}\right)$

(c) Find the equation of BX. Compare the gradient of BX to the gradient of AC.

$m_{BX} = \frac{\frac{1}{2} - 4}{\frac{1}{2} - 1} = \frac{-\frac{7}{2}}{-\frac{1}{2}} = 7$

$m_{AC} = \frac{0-1}{4--3} = -\frac{1}{7}$

$\frac{y-4}{x-1} = 7$

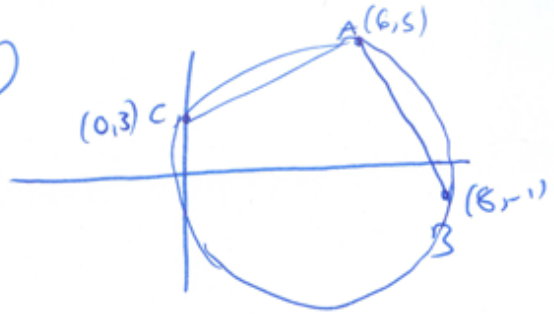
$y - 4 = 7x - 7$

$-4 + 7 = 7x - y$

$\boxed{7x - y = 3}$

Gradients are negative reciprocals (lines are perp.)

(6)



AC

$m = \frac{3-5}{0-6}$

$m = \frac{-2}{-6} = \frac{1}{3}$

$M\left(\frac{0+6}{2}, \frac{3+5}{2}\right)$

$= (3, 4)$

Perp. bisector:

$-3 = \frac{y-4}{x-3}$

$-3x+9 = y-4$

$13 = 3x+y$

AB

$m = \frac{-1-5}{8-6}$

$m = \frac{-6}{2}$

$m = -3$

$M\left(\frac{6+8}{2}, \frac{5+-1}{2}\right)$

$= (7, 2)$

Perp bisector:

$\frac{1}{3} = \frac{y-2}{x-7}$

$x-7 = 3y-6$

$x-3y = 1$

Intersection Point of Perp. bisectors:

$3x+y = 13 \rightarrow 9x+3y = 39$

$x-3y = 1 \rightarrow x-3y = 1$

$10x = 40$

$x = 4$

$3(4)+y = 13$

$y = 1$

Centre is $(4, 1)$

(7)(a)

$3x - y = -6$

$y = 3x + 6$

$m_1 = -\frac{1}{3}$ $(5, 2)$

$-\frac{1}{3} = \frac{y-2}{x-5}$

$-x + 5 = 3y - 6$

$x + 3y = 11$

Intersection pt.:

$3x - y = -6$

$x + 3y = 11$

$\rightarrow$ $9x - 3y = -18$

$x + 3y = 11$

$10x = -7$

$x = -\frac{7}{10}$

$3\left(-\frac{7}{10}\right) - y = -6$

$-y = -6 + \frac{21}{10}$

$-y = -\frac{39}{10}$

$y = \frac{39}{10}$

distance between $(5,2)$ and $\left(-\frac{7}{10}, \frac{39}{10}\right)$

$$d = \sqrt{\left(-\frac{7}{10} - 5\right)^2 + \left(\frac{39}{10} - 2\right)^2}$$

$$d = \sqrt{\left(-\frac{57}{10}\right)^2 + \left(\frac{19}{10}\right)^2}$$

$$d = \sqrt{\frac{3249}{100} + \frac{361}{100}}$$

$$d = \sqrt{\frac{3610}{100}}$$

$$d = \frac{\sqrt{3610}}{10}$$

$$d = \frac{\sqrt{361 \cdot 10}}{10}$$

$$d = \frac{19\sqrt{10}}{10}$$

(b) $3x+4y=9$

$4y = -3x+9$

$y = -\frac{3}{4}x + \frac{9}{4}$

perp. line $m = \frac{4}{3}$ $(-2, 5)$

$\frac{4}{3} = \frac{y-5}{x+2}$

$4x+8 = 3y-15$

$4x-3y = -23$

Intersection Point:

$3x+4y=9 \longrightarrow 9x+12y=27$

$4x-3y=-23 \longrightarrow \underline{16x-12y=-92}$

$25x = -65$

$x = -2.6$

$3(-2.6)+4y=9$

$-7.8+4y=9$

$4y = 16.8$

$y = 4.2$

distance between $(-2,5)$ and $(-2.6, 4.2)$

$d = \sqrt{(-2.6-(-2))^2 + (4.2-5)^2}$

$d = \sqrt{0.36+0.64}$

$d = 1$